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SUB: ECONOMICS
M.A / M.Sc Sem II

COURSE NO: EW-201

GROUP: B (BASIC ECONOMICS)

TOPIC: AUTOCORRELATION

MATERIAL NO: 01

KEY WORDS: AUTOCORRELATION, D-W TEST



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$E(u_t u_{t-1}) \neq 0$ Autocorrelation

Omitted explanatory variables — quasi autocorrelation ✓

Miss-specification of the math. form of the model ✓

Interpretation in the stat. obs. (Smoothing process) ✓

Miss-specification of the true random error term u_t ✓

(war, drought, strike etc continues for more than one period. Hence its effect will be on u_t ✓

— for more than one period. Hence the successive error terms may be correlated. ✓

Consequences of Autocorrelation : —

$E(\hat{\beta}) = \beta$ estimates → biased.

OLS parameters will have higher variances. ✓

(Min-var prop., BEST prop will not be satisfied) ✓

$\text{Var}(u_i)$ may be underestimated ✓

Prediction will be inefficient ✓

(based on estimate of OLS technique) ✓

$$r = \frac{\text{Cov}(x, y)}{\sigma_x \sigma_y} = r$$

$$\text{Cov}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n}$$



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Test of Autocorrelation

Durbin - Watson Test :-

$$d = \frac{\sum_{t=2}^n (\hat{U}_t - \hat{U}_{t-1})^2}{\sum_{t=1}^n \hat{U}_t^2}$$

\hat{U} : OLS estimated residual vector.

$$= \frac{\sum_{t=2}^n (\hat{U}_t^2 + \hat{U}_{t-1}^2 - 2\hat{U}_t\hat{U}_{t-1})}{\sum_{t=1}^n \hat{U}_t^2}$$

\hat{U}_t = estimated OLS residuals.

For large samples ($n > 30$)

$$\sum_{t=1}^n \hat{U}_t^2 = \sum_{t=2}^n \hat{U}_t^2 + \hat{U}_1^2 = \sum_{t=2}^n \hat{U}_{t-1}^2 + \hat{U}_1^2$$

$$d = \frac{2 \sum_{t=1}^n \hat{U}_t^2 - 2 \sum_{t=1}^n \hat{U}_t \hat{U}_{t-1}}{\sum_{t=1}^n \hat{U}_t^2} = 2 - 2 \frac{\sum_{t=1}^n \hat{U}_t \hat{U}_{t-1}}{\sum_{t=1}^n \hat{U}_t^2}$$

$$d = 2(1 - \hat{\rho})$$

Again $-1 \leq \hat{\rho} \leq +1$

$$\hat{\rho} = \frac{\sum \hat{U}_t \hat{U}_{t-1}}{\sum \hat{U}_t^2}$$

- $0 \leq d \leq 4$ when $\hat{\rho} = 0$, $d = 2 \rightarrow$ zero autocorrelation
- $\hat{\rho} = +1$, $d = 0 \rightarrow$ +ve
- $\hat{\rho} = -1$, $d = 4 \rightarrow$ -ve



Sampling distⁿ of d-statistic.
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Total not done
 only du de
 upper and lower
 limits.

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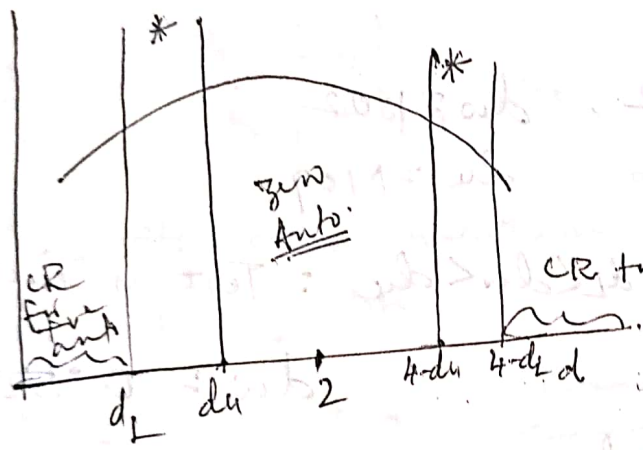
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And tabulated by them.

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d depends on
 a) data matrix (X), b) no. of regressor, k, c) no. of observations, n.
CRITICAL REGION of d statistic

f(d)

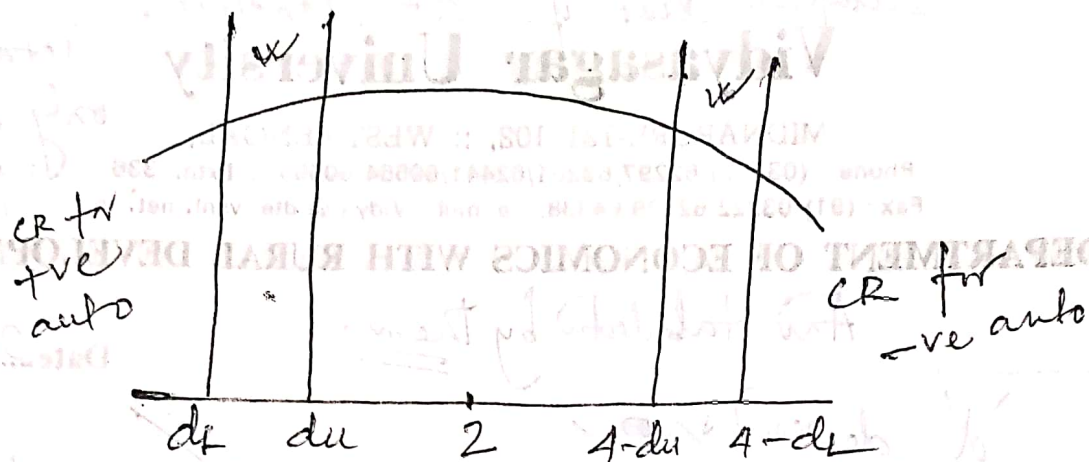


* Inconclusive region.
 $\hat{\rho} = +1 \quad d = 0 \quad +ve \text{ auto}$
 $\hat{\rho} = -1 \quad d = 4 \quad -ve \text{ auto}$
 $\hat{\rho} = 0, \quad d = 2.$

At $n = 16, k = 2, DW = 1.02$ Table $D_L = 0.66, D_U = 1.109$
 Here $D_L < DW < D_U$: Test is inconclusive

If $DW = 1.81$ or $DW = 0.50$ Then see the test. ok

$$d = \frac{\sum_{t=2}^n (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^n \hat{u}_t^2} = 2(1 - \hat{\rho})$$



$$d = \frac{\sum_{t=2}^n (\hat{U}_t - \hat{U}_{t-1})}{\sum_{t=1}^n \hat{U}_t}$$

$$n = 16, \quad k = 2, \quad dw = 1.02$$

$$dL = 0.66, \quad du = 1.09$$

$dL < dw < du$: Test is inclusive

$$\text{if } dw = 1.81$$

$$dw = 0.50$$

$$\text{Then } 4 - dL = 4 -$$

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REMEDIES :-

✓ a) If due to omitted vars : Then try to include those vars.

$$C_t = f(Y_t, Y_{t-1})$$

Then $C_t = a + b_0 Y_t + b_1 Y_{t-1} + u_t$: Then watch the value of DW. Take decision.

✓ b) : Try with other mathematical forms and watch the value of DW.

✓ c) $Y_t = \alpha + \beta X_t + u_t$ Transformation of the model.

$$Y_t^* = \alpha + \beta X_t^* + u_t$$
 Transformed model.

Then apply OLS and get the values of the coefficients.

$$\rightarrow E(u_t u_{t-1}) = 0, \text{ apply OLS.}$$

u_t satisfies the assumption of OLS

and we can apply the OLS technique

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