MPhil

Semester-II

Paper: Eco-121

Advanced Micro Economics: Theory and Applications

Group-B

Lecture-V

NASH EQUILIBRIUM

Existence of Nash Equilibrium

Not every strategic game has the NE, as the game matching pennies show. The condition under which the set of Nash Equilibria of a game is non-empty have been investigated extensively. We are now presenting an existence result.

An existence result has two purposes:

Firstly, if we have a game that satisfies the hypothesis of the result then we know that there is some hope that our efforts to find equilibrium will meet with success.

Secondly, it is the most important purpose. The existence of an equilibrium shows that the game is consistent with a steady state solution.

Further the existence of equilibria of a family of game allows us to study properties of these equilibria without finding them explicitly and without taking the risk that we are studing the empty set.

Mixed Strategy Nash Equilibrium

In the mixed strategy the nature of strategic interaction is such that each player wants to choose that strategy which is not predictable in advance by other player. Consider, for example the matching pennies game. Here it is clear that neither player wants the other player to be able to predict his choice accurately. Thus, it is natural to consider a random strategy of playing heads with some probability p_h and tails with some probability p_t. Such a strategy is called mixed strategy. Strategies in which some choice is made with probability 1 are called pure strategy. If *R* is a set of pure strategies available to Row, the set of mixed strategies open to Row will be the set of all probability distributions over *R*, where the probability of playing strategy *r* in *R* is p_r . Similarly, p_c will be the probability that column plays some strategy *c*. In order to solve the game, we want to find a set of mixed strategies (p_r , p_c) that are, in some sense, in equilibrium. It may be that some of the equilibrium mixed strategies assigns probability 1 to some choice, in which are they are interpreted as pure strategies. The natural starting point in a search for a solution concept is ***standard decision theory:***

We assume that each player has some probability beliefs about the strategies that the other player might choose and that each player chooses the strategy that maximises his expected payoff.

Suppose for example that the payoff to Row is $U_r(r,c)$ if row plays *r*and column plays *c*. We assume that Row has a subjective probability distribution over column's choice which we denote by Π_c . Here Π_c is supposed to indicate the probability. As envisioned by Row, that column will make the choice c. Similarly, Column has some beliefs about Row's behaviour that we can denote by Π_r . We allow each player to play a mixed strategy and denote Row's actual mixed strategy by p_r and Column's actual mixed strategy by p_c . Since Row makes it choice without knowing Column's choice, Row's probability that a particular outcome (*r,c*) will occur is $p_r * \Pi_c$. This is simply the (objective) probability that Row plays *r* times Row's (subjective) probability that column plays *c*. Hence, Row's objective is to choose a probability distribution (p_r) that maximises

Rows' expected payoff = $\sum_r \sum_c p_r \Pi_c U_r(r,c)$

Column, on the other hand, wishes to maximises

Column's expected payoff = $\sum_c \sum_r p_c \Pi_r U_c(r,c)$

Example-1: Calculating Nash equilibrium

	Left	Right
Top	2,1	0,0
Bottom	0,0	1,2

Figure-1