MPhil

Semester-II

Paper: Eco-121

Advanced Micro Economics: Theory and Applications

Group-B

Lecture-V

NASH EQUILIBRIUM

A Nash equilibrium (NE) is a **certain kind of rational expectation equilibrium**.

A NE consists of probability beliefs (π_r, π_c) over strategies and probability beliefs (π_r, π_c) over strategies and probability of choosing strategies (P_r, P_c), such that-

1. The beliefs are corrected $P_r = \pi_r$ and $P_c = \pi_c$ for all r and c, and
2. Each player is choosing (P_r) and (P_c) so as to maximise his expected utility given his beliefs.

A NE is an equilibrium in actions and beliefs. A Ne in pure strategies is a pair (r^*, c^*) such that $U_r(r^*, c^*) \geq \mathrm{U}_r(r, c^*) \forall$ Row strategies *r* and $U_c(r^*, c^*) \geq \mathrm{U}_r(r^*, c) \forall$ Column strategies.

Definition:

A Nash Equilibrium of a strategic game (N, A_i), $(\geq i)$ is a profile $a^* \varepsilon A$ of actions with the property that for every player $i \varepsilon N$ we have

$(a^*_{-i}, a^*_i) \geq i(a^*_{-i}, a_i) \forall a_i \varepsilon A_i$

Thus for a^* to be Nash equilibrium it must be that no player I has an action yielding an outcome that he prefers to generated when he chooses a^*_i, given every other player j chooses his equilibrium action a^*_j. Briefly, no player can profitably deviate, given the actions of other players.

Examples

Example-1: Bach or Stravinsky

Two people wish to go out together to a concert of music by either Bach or Stravinsky. Their main concern is to go out together, but one person prefers Bach and the other person prefers Stravinsky. Representing the individuals' preferences by payoff functions, we have the game in figure-1.

	Bach	**Stravinsky**
Bach	2,1	0,0
Stravinsky	0,0	1,2

Figure-1

This game is also referred to as the "Battle of the Sexes". Bos models a situation in which **players wish to coordinate their behaviour, but have conflicting interests**. The game has two Nash equilibria: (Bach, Bach) and (Stravinsky, Stravinsky). That is, there are two steady states: one in which both players always choose Bach and one in which they always choose Stravinsky.

Example-2: The Prisoner's Dilemma

To suspects in a crime are put into separate cells. If they both confess, each will be sentenced to three years in prison. If only one of them confesses, he will be freed and used as a witness against the other, who will receive a sentence of four years. If neither confess, they will both be convicted of a minor offense and spend one year in prison. Choosing a convenient pay-off representation for the preferences, we have the game in figure-2.

	Non-confess	**Confess**
Non-confess	1,1	4,0
Confess	0,4	3,3

Figure-2: The Prisoner's Dilemma

This game in which there are gains from cooperation- the best outcome for the player is that neither confesses- but each player has an incentive to be a "*free rider*". Whatever one player

does, the other player confesses to Non-confess, so that the game has a unique Nash equilibrium (Confess, Confess).

Do yourself:

Problem-1: Hawk –Dove

	Dove	**Hawk**
Dove	3,3	1,4
Hawk	4,1	0,0

Figure-3: Hawk- Dove

Problem-2: Mozart- Mahler

	Mozart	**Mahler**
Mozart	2,2	0,0
Mahler	0,0	1,1

Figure-4: The Prisoner's Dilemma

Example-3: Matching Pennies

Each of two players chooses either Head or Tail. If the choices differ, person-1 pays Preson-2 a rupee; if they are the same, Person-2 pays Person-1 a rupee. Each person cares only about the amount of money that he receives; a game that models this situation is shown in figure-5. Such a game, in which the interests of the players are diametrically opposed, is called "*strictly competitive*".

	Head	**Tail**
Head	1,-1	-1,1
Tail	-1,1	1,-1

Figure-5: Matching Pennies

The game matching pennies has no Nash equilibrium.