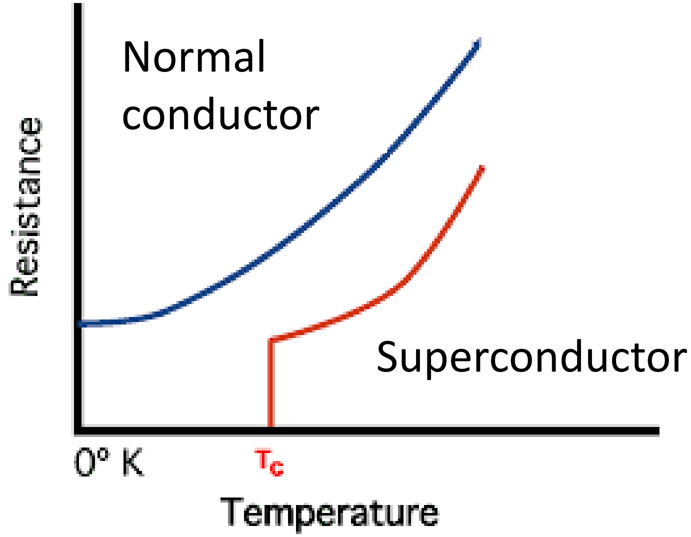
Superconductivity

Kamerlingh Onnes in 1911 observed that the resistance of a sample of Hg drops abruptly at a critical temperature T_c called transition temperature.



In a normal conductor in pure form the resistivity expected to decreases gradually to zero value with the fall of temperature. But in a superconductor while cooling , the resistivity drops abruptly to zero at a certain temperature known as transition temperature. So far 27 elements ,numerous alloys ,ceramic elements (containing CuO) and organic compounds have been discovered to posses superconductivity superconductor having T_c above 77 K are particularly interesting because they don't require liquid helium or hydrogen (b.p. 20K) for cooling.

Superconductivity

Two characteristics are important for Superconductor.

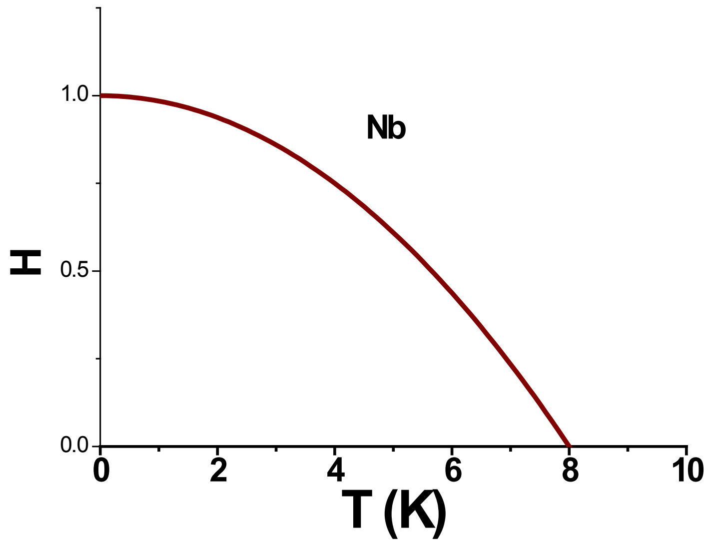
1.Critical field

2. Critical Temperature

The superconducting state of a metal exists only in a particular range of temperature and field strength. Superconductivity will disappear if the temperature of the specimen is raised above its T_C or if a sufficiently strong magnetic field is employed.
The critical value of the applied magnetic field necessary to restore the normal resistivity from super conductivity is a function of temperature.

At critical temperature the critical field is zero i.e. $H_C(T)=0$

Superconductivity



Critical temperature for Niobium(Nb) is shown in figure. We note that the curves are nearly parabolic can reasonably represented by

$$H_C = H_0\left(1 - \frac{T^2}{T_C^2}\right)$$

H_o is the critical field at 0 K. The diagram looks like phase diagram. Inside the curve say (Nb) is in the superconducting phase and outside the curve the material is in the normal phase. The above equation is the equation of phase boundary.

A.C. Resistivity

The fact that a superconducting metal has no resistance means that there is no voltage drop along the metal where a current is passed through it and no power is generated by the passage of current , This however is only strictly true for a direct current of constant value. If the current is charging an electric field is developed and some power is dissipated.

In the range $0<T<T_C$ the conduction electron divide into two class, some behaving as super electrons which can pass through the metal without resistance ,the remainder behaving as normal electrons and can be scattered just like conduction electron in metals. The fraction of super electron appears to decrease as the temperature is raised towards the transition temperature. At 0K all conduction electrons behave like super electrons.

Eventually at transition temperature all the electron have become normal electrons and the metal looses its superconducting properties. Hence a superconducting below its transition temperature can be represented by two electron fluids are normal electron and other super electron

This will be noting that if the current is to remain constant then must be no electric field in the metal ,otherwise the super electrons would be accelerated continuously in this and the current would increase indefinitely.

If there is no field there is nothing to drive the normal currents .

We see therefore that for a constant value of total current all the current is carried by super electrons.

If we apply an alternating field the super electron will therefore lag behind the field because of inertia of super electrons. Hence super electrons will produce an inductive impedance and because that now is an electric field present some of the current will be carried by normal electron. The current is not carried entirely by superconducting electron as in the d.c. case.

The mass of the electron is however extremely small so the inductance due to the inertia is also extremely small. The inductance in Henry of typical superconductor due to inertia of the superconducting electron is only about 10^{-12} of its normal resistance in ohm. So at 1000 Hz only about 10^{-8} of the total current is carried by normal electron and there is only a minute dissipation of power.

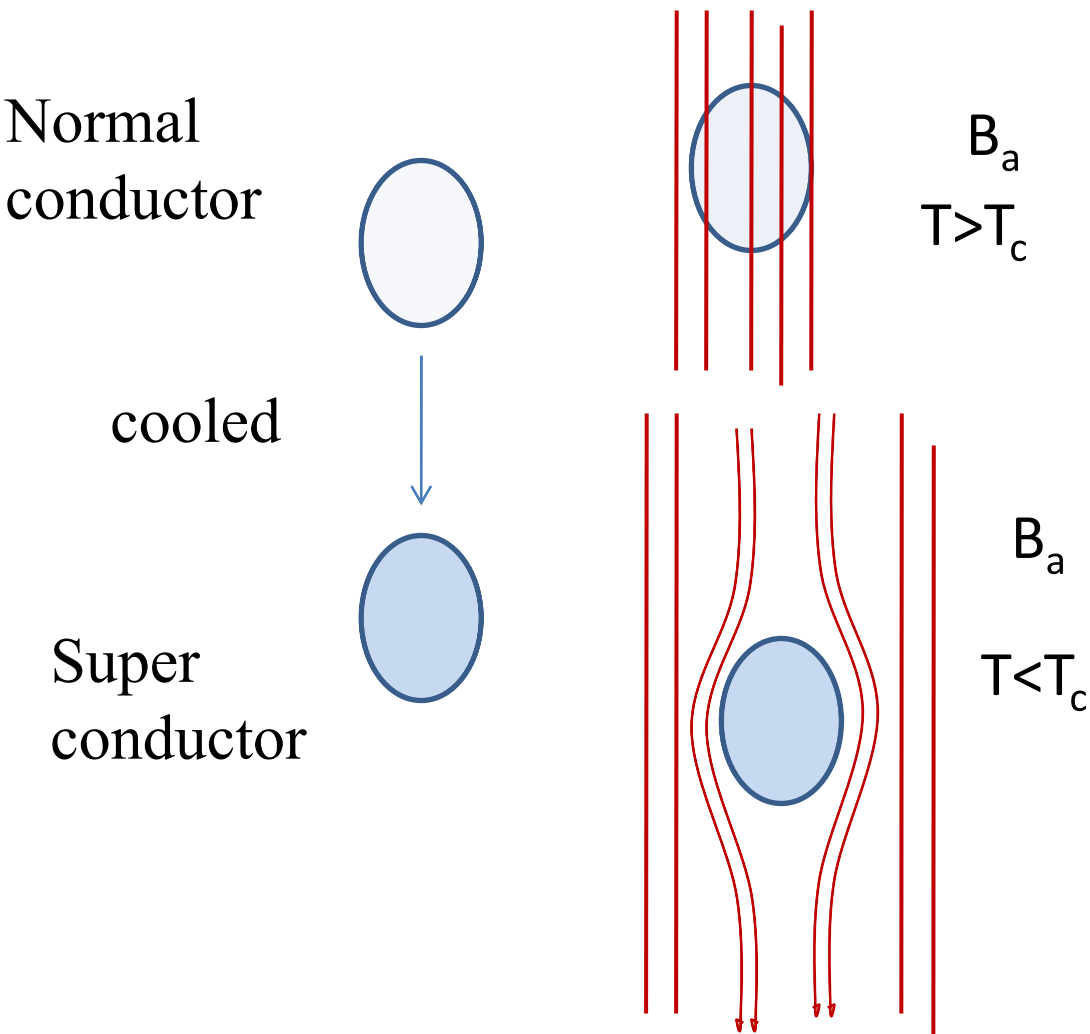
Critical Currents

The magnetic field which causes a superconductors to become normal from a superconducting state is not necessary an external applied field ,if may arise a result of electric current flow in the conductor.

The minimum current that can be raised in a sample without destroying its superconductivity is called critical currents(I_C).If a wire of radius r of a superconductor carries a current I, there is a surface magnetic field $H = \frac{I}{2\pi r}$ associated with the current. If H_I exceeds H_C the material will go to normal.

Meissner effect

Meissner found that a super conductor is cooled in a magnetic field to below the transition temperature, Then at the transition , the lines of induction B are pushed out. This phenomenon is Meissner effect.



B_a changes from 0 to B_a over a time period and induces current which is opposite to the magnetic field.

Meissner effect shows that a superconductor behaves as if inside the specimen.We have B=0.Then the flux density in magnetic material is related to the strength of applied magnetic field.

$B=\mu_0(H_a+M)$

The magnetisation of a superconductor in which B=0

$M=-H_a$

χ is magnetic susceptibility

$X = M_a/H_a=-1$

Superconductor exhibits a perfect diamagnetism

We thus note that the condition of superconducting state are

E=0 (from absence of resistivity)

B=0 (from Meissner effect)

Before Meissners discovery Maxwell equation were thought to be quite adequate to explain the phenomenon of superconductor

Superconductivity

From Maxwells equation we have

$$\nabla \times E = -\frac{dB}{dt}$$

From Ohm's law E=ρj

We see that if the resistivity goes to zero while j is held finite then E must be zero. This E=0 inside a super conductor. From equation 1 We have

B=constant

$$\frac{dB}{dt} = 0$$

Which predicts that the flux through the metal can not change on cooling through the transition .But Meissner showed that a charge does occur as the material passes over to the superconductivity phase. Meissiner showed that magnetic flux is ejected out as soon as the material passes over to the superconductivity phase .

Therefore Meissner effect contradicts the Maxwells equations and suggests that perfect diamagnetism is an essential property of the superconductivity.

London Equation

(Magnetic flux in a perfect conductor)

In a perfect lattice the equation of motion of superconducting electron

$$m\frac{dv}{dt} = eE$$

$$\frac{dv}{dt} = \frac{eE}{m}$$

$$en_s\frac{dv}{dt} = \frac{e^2En_s}{m}$$

$$J = en_s v$$

$$\frac{dJ}{dt} = en_s\frac{dv}{dt}$$

$$\frac{dJ}{dt} = \frac{e^2En_s}{m}$$

$$E = \frac{m}{e^2n_s}\frac{dJ}{dt}$$

Taking curl of the equation

$$\nabla\times E = \frac{m}{n_se^2}\nabla\times\frac{dJ}{dt}$$

$$-\frac{\partial B}{\partial t} = \frac{m}{n_se^2}\nabla\times\frac{dJ}{dt}$$

$$-\dot{B} = \frac{m}{n_se^2}\nabla\times\dot{J}$$

$$= \frac{m}{n_se^2}\nabla\times\frac{1}{\mu_0}(\nabla\times\dot{B})$$

$$-\dot{B} = \frac{m}{\mu_0n_se^2}\nabla\times(\nabla\times\dot{B})$$

$$= \frac{m}{\mu_0n_se^2}(-\nabla^2\dot{B})$$

$$-\nabla^2\dot{B} = \frac{m}{\mu_0n_se^2}\dot{B}$$

$$= \frac{1}{\lambda^2}\dot{B}$$

We have deduced that above behavior by applying usual laws of electromagnetics to a conductor with zero resistance. Solution of the equation

$$\dot{B} = B_0 e^{\frac{\pm x}{\lambda}}$$

+ sign is unphysical as x increases $\dot{B}$ increases

λ is penetration depth $\lambda = \sqrt{\dfrac{m}{\mu_0 n_s e^2}}$

This is differential equation is deduced by the usual laws of electromagnetism to a conductor with zero resistances. Though the treatment completely describes the magnetic properties of a perfectly conductor, it does not adequately describe the behavior of superconductor . Thus Meissner effect shows that inside superconductor the flux density is not only constant but the value is always zero. So not only $\dot{B}$but B itself must die away rapidly below the surface. Hence in superconductor the equation

Superconductivity

$$\nabla^2 B = \frac{B}{\lambda^2}$$

$$\nabla \times B = \mu_0 (J + \frac{\partial D}{\partial t}) = \mu_0 J$$

$$\nabla \times \nabla \times B = -\nabla^2 B = \mu_0 (\nabla \times J)$$

$$\nabla^2 B = -\mu_0 (\nabla \times J)$$

$$-\mu_0 (\nabla \times J) = \frac{B}{\lambda^2}$$

$$-\mu_0 (\nabla \times J) = B \frac{\mu_0 n_s e^2}{m}$$

$$\nabla \times J = -\frac{n_s e^2}{m} B$$

$$B = B_a e^{-x/\lambda}$$