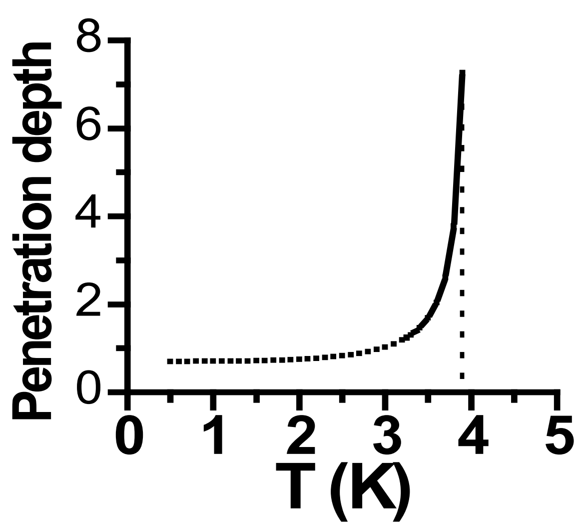
The penetration depth

As the superconductor behaves like a perfectly diamagnet ,B is zero inside the specimen, as B=0, $\nabla \times B = \mu_0 J$ implies J=0 inside the bulk of the specimen. Hence all the current (driving and induced)flow only on the surface. As current flows on the surface, the surface is expected to have a thickness. If the thickness of the surface is zero the current density of the surface becomes infinite and unphysical.

Since $B = B_a e^{-x/\lambda}$ The applied field does not suddenly drop zero at the surface of the superconductor but decays exponentially according to the above equation

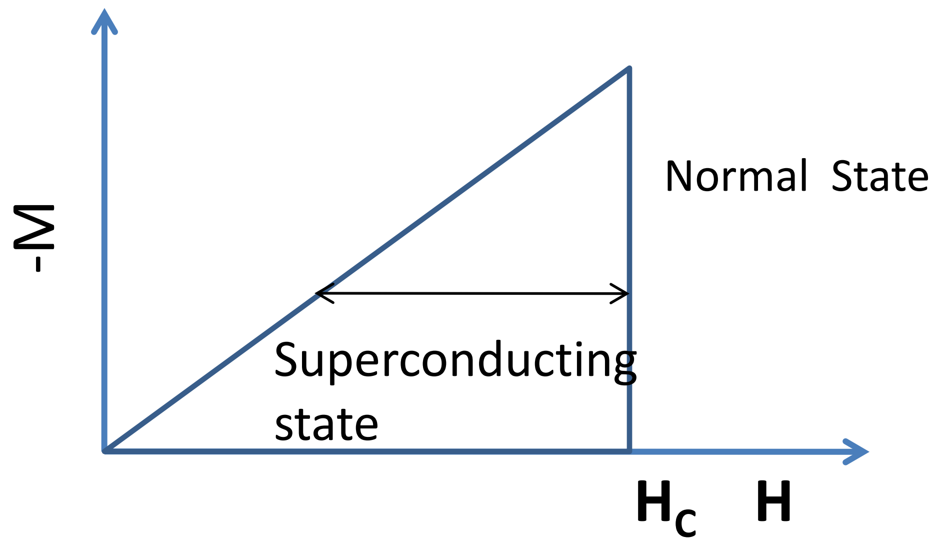
The magnetic field and the surface current are likely to penetrate a superconductor to a depth of 10^{-5} to 10^{-6} cm.

The penetration depth does not have a fixed value but varies with temperature. At low temperature it is nearly independent of temperature ,however the penetration depth increases rapidly and approaches infinity as the temperature approaches the transition temperature.

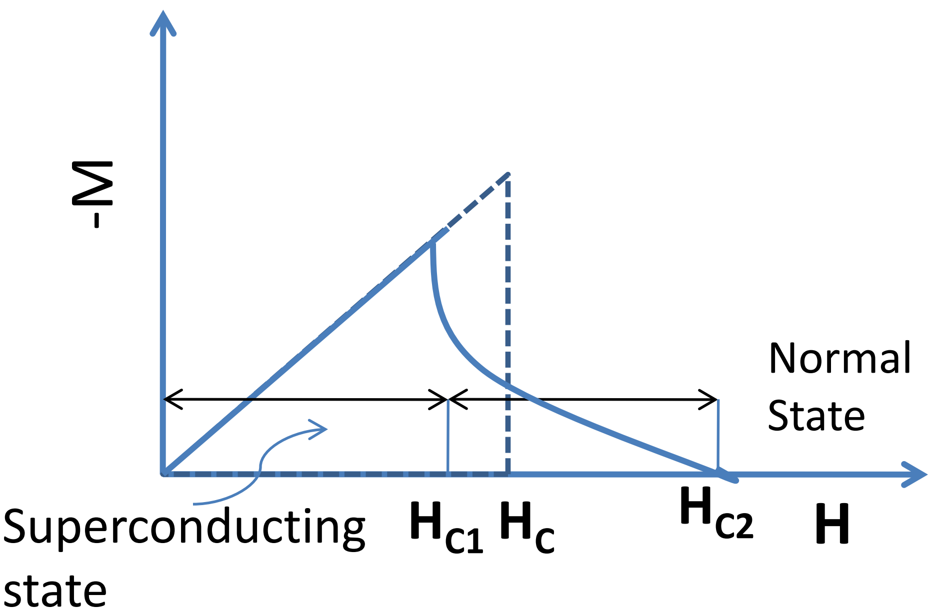


Type I and II Superconductor

For one group of superconductors the magnetization curve is shown in figure. They are completely diamagnetic and hence the flux is completely excluded. These superconductors are type I superconductors or soft superconductor



They are usually pure specimen of some elements and the values of the H_C for them are always too low. They are always to have any technical application in coils. It is quite disappointing that the critical fields of superconductors are so low(H_C is typically 0.1 T or low) because one of the early hopes was to use superconducting wires to produce high magnetic fields needed in many application .It turn out however that there is another class of superconductors which can exist in a mixed state



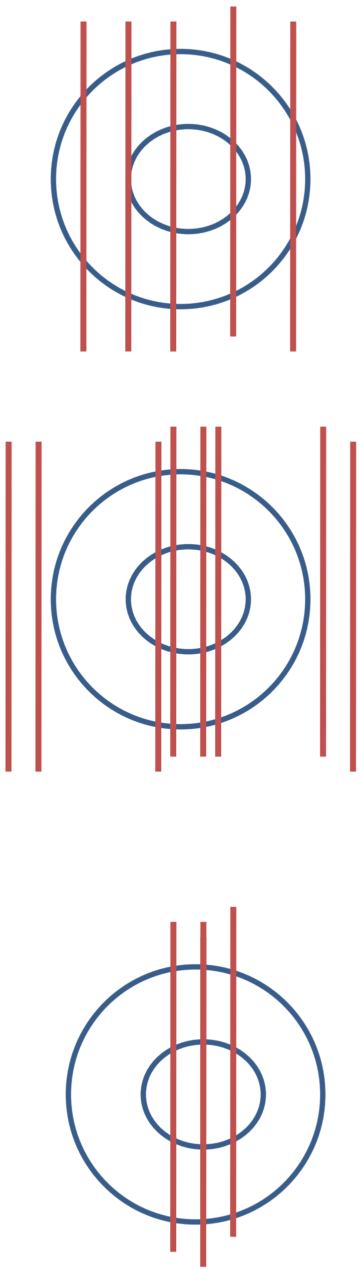
Type II superconductor are a new kind of magnetic phase. They are characterized by lower critical field H_{C1} at which magnetic flux begins to enter the superconductor at an upper critical magnetic field H_{C2} at which superconductivity disappears. The magnetic phase diagram of a type II .

It turns out that the new copper oxide superconductors are extreme type II material with an H_{C2} of the order of 150 T.
A material can change from type I to type II on the substitution of some impurities. Lead is a type I superconductor and it becomes a type II superconductor when two weight percent indium is added to that.

The mixed state is a mixture of normal and superconducting regions ,called the intermediate state. If the field is increased further ,the proportion of normal region grows at the expense of superconducting region and when the external field equal to H_{C2} all over the surface, the specimen is wholly normal and the transition is complete

How to explain persistant current setup in side superconductor according to Meissner effect.

The flux at transition temperature is ejected due to the induction of current in the specimen which produces a flux equal and opposite to the original flux due to the magnetic field.



Since in the superconducting state resistivity zero the current due to induction in the specimen persists as long as the field is on. But the persistent current arise in the absence of field.

Suppose the specimen in the form of anchor ring and placed in a magnetic field .The flux will penetrate all space .Now the temperature of specimen is reduced down to the transition temperature. Consequently specimen due to occurrence of superconductor state eject the flux. Now the applied field is removed. The magnetic flux inside the hollow circular space of the ring will be trapped in it because it can not cut through this ring which has acquired a property of diamagnetic superconductor.

Flux inside the circular space is trapped and do not collapse. Obviously the persistent current in the specimen maintain the flux through the hole. This explain that irrespective of the presence or absence of applied field persistent currents are maintained in the superconductor.

Super conductor in a.c. field

The behavior of super conductors in d.c. field is given by the equation

$$\nabla \times J = -\frac{n_s e^2 B}{m} = -\frac{\mu_0 n_s e^2 H}{m}$$

Now ,Maxwells equation

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$= J_n + J_s + \frac{\partial D}{\partial t} \qquad 1$$

$$J_s = -e n_s v_s$$

Now

$$\frac{dJ_s}{dt} = -e n_s \frac{dv_s}{dt}$$

But $m\frac{dv_s}{dt} = eE$

$$= \frac{e^2 n_s}{m} E$$

This is London equation which describes the absence of resistance

The equation also shows that it is possible to have steady currents in absence of electric field (for E=0,J_s is finite and steady).The corresponding expression for normal current density is $J_n=\sigma_n E$ which shows that no current is possible in absence of an electric field.

$$\frac{dJ_s}{dt} = \frac{E}{\mu_0 \lambda^2}$$ 2

Let us take the field and the current to be sinusoidal

$J_s = J_0 e^{jwt}$ $H = H_0 e^{jwt}$

$$\frac{dJ_s}{dt} = j\omega J_s$$ 3

Combining (2) and (3)

$$j\omega J_s = \frac{E}{\mu_o \lambda^2}$$

$$J_S = \frac{1}{j\omega} \frac{E}{\mu_o \lambda^2} = -\frac{jE}{\omega \mu_0 \lambda^2}$$ 4

Further

$$D = D_0 e^{j\omega t}$$

$$\frac{\partial D}{\partial t} = j\omega D = j\omega \varepsilon E$$ 5

Also $J_n = \sigma_n E$ 6

Combining equation (4),(5) and (6)with equation (1) we get

Superconductivity

$$\nabla \times H = (\sigma_n + j\omega\varepsilon - \frac{j}{\omega\mu_0\lambda^2})E$$

Taking curl on both sides

$$\nabla \times \nabla \times H = (\sigma_n + j\omega\varepsilon - \frac{j}{\omega\mu_0\lambda^2})\nabla \times E$$

$$= \left(\sigma_n + j\omega\varepsilon - \frac{j}{\omega\mu_0\lambda^2}\right)\left(-\frac{dB}{dt}\right)$$

$$= \left(\sigma_n + j\omega\varepsilon - \frac{j}{\omega\mu_0\lambda^2}\right)\left(-\mu_0\frac{dH}{dt}\right)$$

$$= \left(\sigma_n + j\omega\varepsilon - \frac{j}{\omega\mu_0\lambda^2}\right)\left(-\mu_0 j\omega H\right)$$

$$\nabla^2 H = j\omega\mu_o\left(\sigma_n + j\omega\varepsilon - \frac{j}{\omega\mu_0\lambda^2}\right)H$$

$$\nabla^2 H = \left(\frac{1}{\lambda^2} - \omega^2\mu_0\varepsilon + j\omega\mu_0\sigma_n\right)H$$

Where H/λ^2 represents the field penetration due to supercurrent, $-\omega^2\mu_0\varepsilon H$ represents the field penetration due to displacement currents. $\omega\mu_0\sigma H$ represents field penetration due to eddy current

1) For a d.c. field $\omega=0$ $\nabla^2 H = \frac{H}{\lambda^2}$

This is the same as predicted by London theory.

2) At microwave frequencies ω is sufficiently high or the wave length λ_0 is small,

$$\omega^2\mu_0\varepsilon = \frac{\omega^2}{c^2} = \frac{4\pi^2}{\lambda_0^2}$$

It is negligible compared to other two terms

$$\nabla^2 H = \left(\frac{1}{\lambda^2} + j\omega\mu_0\sigma_n\right)H$$

Penetration depth and skin depth

From equations (4) and (6)

$$\frac{J_s}{J_n} = \frac{-jE/\omega\mu_0\lambda^2}{\sigma_n E}$$

$$\frac{J_n}{J_s} = \frac{-\sigma_n E\omega\mu_0\lambda^2}{jE}$$

$$= \lambda^2\mu_0\omega\sigma_n \qquad 7$$

$$\frac{\lambda^2}{\delta^2} = \frac{\lambda^2}{2/\mu_0\omega\sigma_n}$$

$$= \frac{\lambda^2\mu_0\omega\sigma_n}{2}$$

$$= \frac{1}{2}\left(\frac{J_n}{J_s}\right) \qquad 8$$

From (7) and (8) we get $$\delta^2 = \frac{2}{\mu_0 \omega \sigma_n}$$

Which predicts that when frequency is sufficiently high, δ is small,

penetration depth λ becomes larger compared to skin depth. With $\lambda > \delta$ we infer that $J_n > J_s$ or normal current predominates. This implies that at high frequency superconductor behaves like normal conductor.