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Barro Growth Model

Introduction

Like the learning by doing approach or knowledge spill over model, Barro (1990) developed the increasing returns to scale in the overall production system by introducing the public sector that is capable of endogenizing the technological progress and thereby explaining the increasing growth of capital and output per capita in the long run. In the Barro model public spending goes for public investment (infrastructures, schools, sanitation, institutional facilities, good governance, etc.). Public investments, which are financed through income taxes, complement private investments so that there are crowding-in effects of this public investment and thereby promoting growth of output. Since public investments raise the productivity of private investments, higher taxes can be associated with an increase or a decrease in overall growth. If government expenditure is kept fixed and there are constant returns to scale in L and K, then the working of diminishing returns to the factors cannot be barred. If we allow government expenditure as variable in accumulation of capital, then the working of diminishing returns will no longer be there and the economy is capable of producing endogenous growth like the simple AK type model.
The model of Barro adds public spending to the AK model. Suppose the production function is like the following-

$$Y = AL^{1-\alpha}.\ K^{\alpha}.\ G^{1-\alpha}$$

where G stands for the public expenditure on goods and services. The production function exhibits increasing returns to scale (IRS). Suppose increase all the factors by λ proportion then new output is

$$A.(\lambda L)^{1-\alpha}.\ (\lambda K)^{\alpha}.\ (\lambda G)^{1-\alpha} = \lambda^{2-\alpha}.\ AL^{1-\alpha}.\ K^{\alpha}.\ G^{1-\alpha} = \lambda^{2-\alpha}.\ Y$$

Since $0<\alpha<1$, $2-\alpha >1$ and hence new output after introduction of government sector is greater than the scale effect. Hence, IRS is working and there is possibility of making increasing growth rates of income over time.

We saw in the Y = AK model, anything that changed the level of baseline technology, A, affected the per capita K and income in the long run that broke the chain of diminishing marginal productivity of K.

In the models of Arrow and Romer we saw the roles of knowledge creation and spill overs (through Learning by Doing and Learning by Investments) to justify A (or to break diminishing MPk) and in Lucas model we saw the role of human capital formation, besides physical capital formation, K) proved the role of A and justified MPk as constant.

In all these models we established the existence of positive growth of per capita income in the long run framework which was an unlikely event in Solow model.

In the Barro model, the replacement of 'A' is done by government intervention where government provides public services like internal security, food security, public infrastructure (like education, health, defence, etc) which are non-rival and non-excludable in nature. Hence, public good or public services **without variable tax and, congestion and corruption**, are another source of the AK form of production function of an economy. Here, government's choices about public services determine the coefficient 'A' and thereby affect the long run growth rate of the economy. **We can have both decentralized and social planner's solution like Ramsey model.**

The model for Decentralized Objectives by Households and Firms

Suppose there is G as the additional input (to represent public services or **public input**) with L and K (**as private inputs**). The Cobb Douglas production function for firm 'i' is-

$Y_i = AL_i^{1-\alpha}. K_i^{\alpha}. G^{1-\alpha}$

where $0<\alpha<1$.

The production function for each firm shows the working of CRS in private inputs, L and K (**as the sum of their powers is 1**) and the inclusion of G as another factor of production leads the total production system to IRS (**as the sum of their powers is greater than 1).**

Assume that aggregate labour force is constant. In that case, if G is constant, the returns from private capital (K) will be diminishing since the production function will then be

$Y_i = AL^*. K_i^{\alpha}. G^* = AL^*G^*. K_i^{\alpha} = A^*. K_i^{\alpha}$

Here $dY_i/dK_i = A^*. \alpha. K_i^{1-\alpha}$

And $d^2Y_i/dK_i^2 = - \alpha.(1-\alpha) K_i^{-\alpha} <0$ (as in the Solow and Ramsey models).

On the other hand, if G increases along with K (that means both private and public capitals move in positive directions simultaneously like Crowding in effects) then diminishing MPk will not arise. Here public capital is complementary to K and L and increase in G leads to increase in the MPL and MPk (this is the source of endogenous growth).

This means the production function will exhibit CRS in K and G with a constant labour force L*. For this reason, the economy will be capable of generating endogenous growth as in AK type production function.

Suppose the government finances its purchase of goods and services with lump sum tax, not by any variable tax.

For given G, each profit maximizing firm will optimize the use of K where the equilibrium condition will be satisfied. The equilibrium condition is MPk = rental price = $r + \delta$.

We have the production function as

$Y_i = AL_i^{1-\alpha}. K_i^{\alpha}. G^{1-\alpha}$

In per capita terms-

$y_i = Y_i/L_i = (AL_i^{1-\alpha}. K_i^{\alpha}. G^{1-\alpha})/ L_i = A. L_i^{-\alpha}. K_i^{\alpha}. G^{1-\alpha}$

or, $y_i = A. (K_i^{\alpha}/ L_i^{\alpha}). G^{1-\alpha} = A. (K_i/ L_i)^{\alpha}. G^{1-\alpha} = A. (k_i)^{\alpha}. G^{1-\alpha}$

Hence, $MP_k = \alpha.A. (k_i)^{\alpha-1}. G^{1-\alpha} = r + \delta$

If this marginal condition holds for all the firms then each firm will choose the same k_i = k. Hence the aggregate production function will be-

$\Sigma y_i = y_i{}^*L = Y = AL.\ k^{\alpha}.\ G^{1-\alpha} = AL.\ k^{\alpha}.\ (G/G^{\alpha})$

Or, $G^{\alpha} = (AL.\ k^{\alpha}.\ G)/Y = AL.\ k^{\alpha}.\ (G/Y)$

Or. $G = (G/Y)^{1/\alpha}.\ (AL)^{1/\alpha}.\ k$ ………………. (i)

Now the firms' equilibrium condition becomes (putting the value of G in MPk)-

$r + \delta = MP_k = \alpha.A.\ (k)^{\alpha-1}.\ [(G/Y)^{1/\alpha}.\ (AL)^{1/\alpha}.\ k]^{1-\alpha}$

or, $r + \delta = \alpha.A.\ (k)^{\alpha-1}.\ k^{1-\alpha}\ (G/Y)^{1-\alpha/\alpha}.\ A^{1-\alpha/\alpha}\ L^{1-\alpha/\alpha}$

or, $r + \delta = \alpha.A^{1/\alpha}.\ (G/Y)^{1-\alpha/\alpha}.\ L^{1-\alpha/\alpha} \neq f(k)$

Usually governments of all the economies want to maintain a constant ratio of G/Y (=Govt. Expd/GDP). If G/Y is constant then MP_k (or $r + \delta$) is invariant to per capita capital, k. It is observed from the above equilibrium condition that there is no 'k' term in this relation. So, MP_k cannot diminish as k increases due to the presence of public good or services. Hence, positive growth of per capita GDP can be possible under the Barro model.

Further, it is seen that as L increases MP_k also increases. This is the Scale Effect which was similar to that of Romer and Lucas model.

Comparison with AK model

This constant MP_k parallels the conclusion made by the AK model. The constant MP_k plays the same role in the growth process that the constant A played in the AK model. Hence, there is no transitional dynamics in the Barro model (as MPk is constant).

Thus, the growth rates of per capita consumption (c), per capita capital (k) and per capita income (y) grow at the same constant rate.

Now we can determine that common growth from the consumption growth expression. We had the consumption growth expression from Ramsey model that –

$$\frac{\dot{c}}{c} = 1/\sigma[f_k - n - \delta]$$

Or, the rate of growth of 'c' = $1/\sigma[\alpha.A^{1/\alpha}.\ (G/Y)^{1-\alpha/\alpha}.\ L^{1-\alpha/\alpha} - n - \delta]$

This result shows that rate of growth of consumption is positively related to G/Y since the public services were not financed by non-distorting income tax.

Social Planner's Problem under Lump Sum Tax

In the previous analysis we considered the decentralized behaviour of households and firms under the structure of lump sum tax. 'G' was financed from other sources in place of any variable taxes.

The production function is as before- $y = A.\ (k_i)^{\alpha}.\ G^{1-\alpha} = A.\ k^{\alpha}.\ G^{1-\alpha}$

The common budget constraint faced by the planner is –

$dk/dt = dm/dt = \dot{m} = y - c - \delta k - G/L$ **(G/L = per capita consumption of public goods/services). Here 'm' is the state variable.**

Like in Ramsey model we set the Hamiltonian function for Utility maximization with respect to c, G and k.

$H = u(c_t).\ e^{-(\rho - n)t} + \beta_t\ [y - c - \delta k - G/L]$

Here β_t is the co-state variable. Assuming n = 0 (since L is constant) and the constant elasticity of substitution utility function-

or, $H(c, G, k) = (c^{1-\sigma}-1/1-\sigma).\ e^{-\rho t} + \beta_t[y - c - \delta k - G/L]$

or, $H(c, G, k) = (c^{1-\sigma}/1-\sigma).\ e^{-\rho t} + \beta_t[A.\ k^{\alpha}.\ G^{1-\alpha} - c - \delta k - G/L]$

The FOCs are-

i) $\delta H/\delta c = 0$, >> $c^{-\sigma}.\ (1-\sigma)/(1-\sigma).\ e^{-\rho t} = \beta$ ……… (1)

ii) $\delta H/\delta G = 0$, >> $(1-\alpha).\ \beta.\ A.\ k^{\alpha}.\ G^{-\alpha} - \beta/L = 0$

or, $(1-\alpha).\ A.\ k^{\alpha}.\ G^{-\alpha} = 1/L$ …………….. (2)

iii) $\delta H/\delta k = -\delta\beta/\delta t$ >> $\beta\ .\alpha.\ A.\ k^{\alpha-1}.\ G^{1-\alpha} - \beta\ \delta = -\delta\beta/\delta t$ ………..(3)

And the transversality condition $\underset{t \to \alpha}{Lt} = [\beta_t\ .\ m_t] = 0$

Now take derivative of β w.r.t time from (1) and substitute it into (3).

We get from (1), $-\sigma\ .c^{-\sigma-1}.\ \delta c/\delta t.\ e^{-\rho t} = \delta\beta/\delta t$

Or, $-\sigma\ .c^{-\sigma}.\ [(\delta c/\delta t)/c].\ e^{-\rho t} = \delta\beta/\delta t$

After substitution to (3) we get-

$\beta\ (\alpha.\ A.\ k^{\alpha-1}.\ G^{1-\alpha} - \delta) = -(-).\sigma\ .c^{-\sigma}.\ [(\delta c/\delta t)/c].\ e^{-\rho t}$

Now putting the value of β from (1) into the above relation-

$c^{-\sigma}.\ (1-\sigma)/(1-\sigma).\ e^{-\rho t}\ .(\alpha.\ A.\ k^{\alpha-1}.\ G^{1-\alpha} - \delta) = \sigma\ .c^{-\sigma}.\ [(\delta c/\delta t)/c].\ e^{-\rho t}$

or, $\alpha.\ A.\ k^{\alpha-1}.\ G^{1-\alpha} - \delta = \sigma.\ [(\delta c/\delta t)/c]$

or, $[(\delta c/\delta t)/c] = 1/\ \sigma\ [\alpha.\ A.\ k^{\alpha-1}.\ G^{1-\alpha} - \delta]$ ………… (4)

Now from (2) $(1-\alpha).\ A.\ k^{\alpha}.\ G^{-\alpha} = 1/L$

Or, $(1-\alpha).\ A.\ k^{\alpha}.\ G^{-\alpha}.\ G = G/L$

Or, $(1-\alpha).\ A.\ k^{\alpha}.\ G^{1-\alpha} = 1/L$

Or, $(1-\alpha).\ A.\ L.\ k^{\alpha}.\ G^{1-\alpha} = 1$

Or. $(1-\alpha).\ Y = G$

Or, G/Y = (1-α)

Or, G/Y = (1-α)

This means govt. expd. To GDP (=G/Y) is constant. In other words, the optimal condition of the govt. is to maintain a constant G/Y in the model with lump sum tax.

Now put the expression of G from (i) of the two back pages in (4)-

$(\delta c/\delta t)/c = 1/\sigma\, [\alpha.\, A.\, k^{\alpha-1}.\, G^{1-\alpha} - \delta]$

Or, $(\delta c/\delta t)/c] = 1/\sigma\, [\alpha.\, A.\, k^{\alpha-1}.\, G^{1-\alpha} - \delta]$

Or, $(\delta c/\delta t)/c] = 1/\sigma\, [\alpha.\, A.\, k^{\alpha-1}.\, \{(G/Y)^{1/\alpha}.\, (AL)^{1/\alpha}.\, k\}^{1-\alpha} - \delta]$

Or, $(\delta c/\delta t)/c] = 1/\sigma\, [\alpha.\, A.\, k^{\alpha-1}.\, \{(1-\alpha)^{1-\alpha/\alpha}.\, (AL)^{1-\alpha/\alpha}.\, k^{1-\alpha} - \delta]$

Or, $(\delta c/\delta t)/c] = 1/\sigma\, [\alpha.\, A^{1/\alpha}.\, (1-\alpha)^{1-\alpha/\alpha}.\, L^{1-\alpha/\alpha} - \delta]$

Therefore the social planner's optimum solution is identical to that under decentralized system under the lump sum tax assumption.

The optimal rate of growth of k, y and c will be at the rate of-

$$\frac{\dot{c}}{c} = 1/\sigma[f_k - n - \delta]$$

$\dot{c}/c = 1/\sigma[\alpha.A^{1/\alpha}.\, (G/Y)^{1-\alpha/\alpha}.\, L^{1-\alpha/\alpha} - n - \delta]$

$\quad = 1/\sigma\, [\alpha.A^{1/\alpha}.\, (1-\alpha)^{1-\alpha/\alpha}.\, L^{1-\alpha/\alpha} - n - \delta]$

Hence, as scale factor, L, increases, MPk increases, breaks its diminishing nature and ultimately $\dot{c}/c$ increases.

Introduction of Income tax on Households' Wage and Asset Income, Consumption Tax and Firms' Profit Tax

Let us recall Ramsey model with government sector.

Govt. spends G and make transfer payments V in lump sum amount. Hence, total amount of govt. expenditure is G + V.

Govt. finances this expenditure by means of taxes upon households' Wage and Asset Income, Consumption Tax and Firms' Profit Tax.

Suppose, 'tw' is tax rate on wage income, 'ta' is the rate on asset income, 'tc' is on consumption and 'tf' is on firms' income. Wage income after tax will now be wL-tw.wL = (1-tw).wL. Similarly for asset income, it will be (1-ta)r.M, for consumption, (1-tc).C.

So govt.'s balanced budget equation is-

G + V = tw. w. L + ta. r. M + tc. C + tf. F

Household's Problem

Households' budget constraint which says that change of per head asset (m) over time will be the difference between sum of wage and asset income net of tax and consumption tax and population growth factor plus transfer amount per capita. This is-

$dm/dt = \dot{m} = (1\text{-}tw).\ w + (1\text{-}ta).\ r.\ m - (1 + tc).\ C - n.\ m + v$ where $v = V/L$

Now with the household's utility function, $u(c) = (c^{1-\sigma}\text{-}1/1\text{-}\sigma)$, the consumption growth rate is given as-

$\dot{c}/c = 1/\sigma\ [\underline{(1\text{-}ta)\ r}\ - n]$

That means, household's decision to defer consumption depends on the after tax returns on asset (1-ta)r. The tax rate on consumption and it does not enter the growth of consumption expression.

Firm's Problem

$Y = F(K, L)$, and $R = r + \delta$

Profit before tax = F(K,L)-wL-RK

Profit after tax =∏= (1-tf)[F(K,L)-wL- δ K] – r.K

FOC: d∏/dk = (1-tf)[fk- δ] – r = 0

Or, fk = r/(1-tf) + δ

This means, a higher tf raises the required MPk.

As r increases, fk increases

And, As δ increases, fk increases

And As tf increases, fk increases

Social Planner's Problem

Here we form the Hamiltonian under this variable tax regime and calculate the following consumption growth expression from the FOC conditions-

Now taking the household's constant elasticity utility function as before the growth rate of consumption per capita is rewritten as-

$\dot{c}/c = 1/\sigma\ [\underline{(1\text{-}tw)\ (1 - tf).\ \alpha.A^{1/\alpha}.\ (G/Y)^{1\text{-}\alpha/\alpha}.\ L^{1\text{-}\alpha/\alpha}}\ - n\text{-}\delta]$

The underlined term in the above expression indicates the after tax MPk. It shows that the post-tax MPk is smaller than pre-tax MPk as tw > 0 and tf > 0.

Now the impact of increase in (G/Y) may not unambiguously affect $\dot{c}/c$, $\dot{y}/y$ or $\dot{k}/k$ because now the financing of G is done by variable or distorting income taxes, not by non-distorting lump sum tax.

Therefore, as G/Y increases, $\dot{c}$ /c will increase if the effect of G on $\dot{c}$ /c is not offset by 'tw' or 'tf'. The relation between G/Y and $\dot{c}$ /c will then be non-monotonic-first rising (when tax effect is not dominant) and subsequently falling (when tax effect is dominant).

Assignments for Self-assessment

Questions of 2 Marks

1. What are the differences between a Cobb-Douglas production function and an AK type production function in terms of returns to the variable factors and returns to scale?
2. What are the causes behind divergence among developed and less developed economies during the late sixties to late eighties despite the working of the diminishing marginal productivity of capital?
3. Why economic growth can be termed as endogenous?
4. What is discount rate? Write down a dynamic utility function in an infinite time period incorporating discount factor.
5. Write down a utility function involving consumption having constant elasticity of substitution.
6. Derive consumption elasticity from the utility function u(c) = ($c^{1-\sigma}$-1/1-σ).
7. Mention two important features of public services in respect to their composition and usability.
8. Intuitively explain the relation between income growth and G/Y ratio.
9. What do you mean by Scale Effect under Barro model.
10. Why it is said that the AK model has no transitional dynamics like Solow model?

Questions of 6-10 Marks

1. Explain how steady state solution under Ramsey model is obtained by the help of endogenous savings and consumption behaviour of the households in a decentralised framework.
2. Examine whether the low population policy undertaken by developing countries like China would make positive influence upon growth rate of income. Think in terms of the neoclassical and endogenous growth theories' structures.
3. Explain how positive growth of income, consumption and capital formation are ensured in a production function with government sector without having distorting taxes on income, consumption, assets and firms' earnings.
4. Can you establish that there can be unambiguous positive growth of income in a system with public expenditures financed by income taxes on different economic agents? Discuss.

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