M. Sc. Semester IV' 2020

Subject: Physics

Course No.: PHS 402.1 (Nuclear Physics-II)

Section: Nuclear models

Topic: Nuclear models – Liquid drop model and Bohr-Wheeler theory of nuclear fission

Lecture No. 2 & 3

by

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Lecture No. 2

A. Nuclear models:

A nuclear model is any of several theoretical descriptions of the structure and function of atomic nuclei.

There are many isolated facts which will require explanation when we adopt any nuclear theory. Few of them are:

1. Why do nuclei emit α-particles and β-particles ?
2. Why is the B.E./nucleon almost constant ?
3. Why are the $4n$ nuclei particularly stable ?
4. How do we explain the existence of excited states of nuclei ?
5. How do we explain the Geiger-Nuttal rule ?
6. How do we interpret the special properties of nuclei such as stability, spin, magnetic moment, etc. ?

As we have seen that a nucleus shows a no. of properties. For instance, it has

1. a radius $r \propto A^{1/3}$.
2. a well-defined and uniform density.
3. a B.E./nucleon which is very nearly constant, except for very small A value.
4. a n/p which is close to unity for small A, but progressively increases as A increases.

Apart from these, an atomic nucleus is **involved** in many types of nuclear reactions and **exhibits** some phenomena such as nuclear fission and nuclear fusion.

Nuclear models may be divided into two broad categories:

1. Strong interaction models (SIM) – in which the nucleons of a nucleus are taken to be strongly coupled to one another because of their strong and short-range internucleonic forces.
2. Independent particle models (IPM) – in which the behavior of individual nucleons within the nucleus determines the characteristics of the nucleus as a whole and the nucleons move nearly independently in a common nuclear potential.

B. Liquid drop model:

Historically, the liquid drop model was the first model developed to explain various properties of nuclei. It was put forward to explain the **saturation properties of nuclear interaction** in analogy with the attractive forces between the constituents of a classical liquid. That means this model is a description of atomic nuclei in which the nucleons (neutrons and protons) behave like the molecules in a drop of liquid.

Russian-born American physicist George Gamow formulated the model in 1929. The liquid drop model was applied by C. F. von Weizsacker and H. A. Bethe to develop a **semi-empirical B. E. formula**. Danish physicist Niels Bohr and American physicist John Archibald Wheeler developed this model and used it to **explain nuclear fission** in 1939.

Similarities between the molecules in a drop of liquid and the nucleons in a nucleus:

1. Nuclear attractive forces analogous to surface tension forces.
2. The nucleons behave in a manner similar to that of molecules in a liquid.
3. Heat of evaporation and density of a liquid drop are independent of the type of the drop size, whereas B. E./nucleon and density of a nucleus are independent of the type of nucleus or the mass number A.
4. Bothe liquid drop and the nucleus possess constant density.
5. The constant B.E./nucleon of a nucleus analogous to the latent heat of vaporization of liquid.
6. Disintegration of nuclei analogous to the evaporation of molecules from surface of liquid.
7. Like the molecules, the nucleons interact only with their intermediate neighbours.
8. The internal energy of nuclei corresponds to the internal thermal vibrations of drop molecules.
9. The formation of compound nucleus and absorbing the bombarding particles are correspond to the condensation of drops.

Dissimilarities:

1. Molecules attract one another at distances larger than the dimensions of the electron shells and they repel strongly when the distance is smaller than the size of the electron orbits. On the other hand, nuclear forces are attractive within the smaller range, the range of the nuclear forces.
2. The average K.E. of the molecules in the liquid is of the order of $0.1\ eV$, the corresponding de Broglie wavelength is $5 \times 10^{-11}\ m$. The average K.E. of the nucleons in nuclei is of the order of $10\ MeV$, the corresponding de Broglie wavelength is $6 \times 10^{-15}\ m$, which is of the order of inter-nucleon distances.
3. The motion of the molecules in a liquid is of classical character, whereas in the nuclei the motion of the nucleons is of quantum character.

The liquid drop model **helps explain** nuclear phenomena such as the energetic of nuclear fission and the binding energy of nuclear ground levels.
The Bethe-Weizscaker semi-empirical mass formula is of great practical importance because the masses and hence the binding energies are predicted using the liquid drop model with considerable accuracy in terms of only five parameters – volume B.E., surface B.E., Coulomb energy, asymmetry energy and pairing energy terms.

Although **inadequate to explain** all other nuclear phenomena, the theory underlying the model provides excellent estimates of average properties of nuclei.

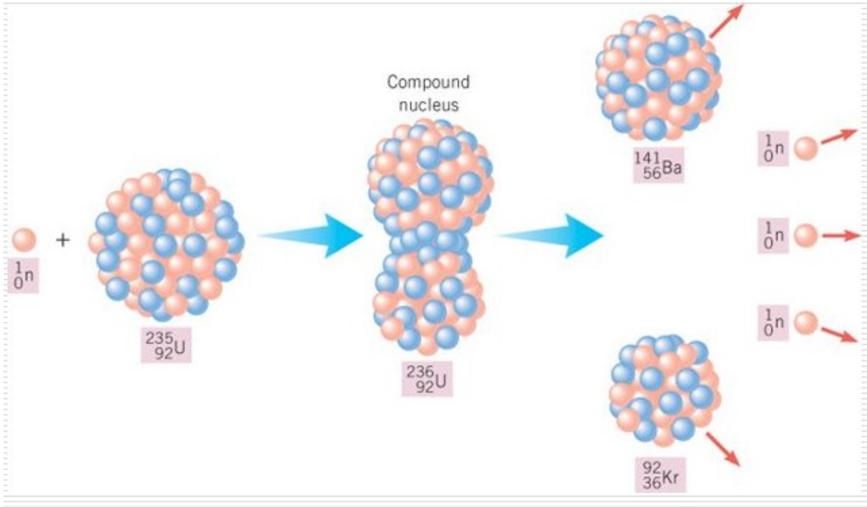
Lecture 3

C. Liquid drop theory of fission (Bohr-Wheeler theory of nuclear fission):

N. Bohr and J. A. Wheeler put forward the theory of nuclear fission in 1939. The theory is based on the liquid drop model of the nucleus. On the basis of this theory, it is possible to calculate the activation energy E_f and critical energy for fission of different nuclei.

If a thermal neutron hits the nucleus, a compound nucleus is formed with certain exciting energy due to the extra neutron. Then this energy sets up the compound liquid drop nucleus into rapid oscillations.

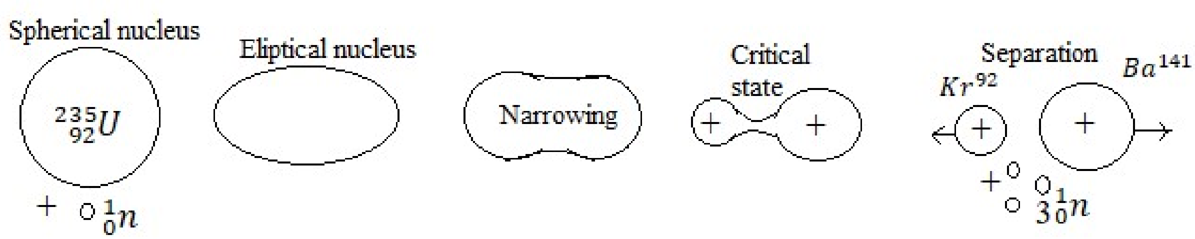
$$^{235}_{92}U + ^{1}_{0}n \rightarrow \ ^{236}_{92}U(compound\ nucleus) \rightarrow \ ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n$$



The vibrations set up in the compound nucleus to deform it due to which its surface energy E_s and Coulomb energy E_c are both changed.
The sum of energies for a **spherical nucleus**

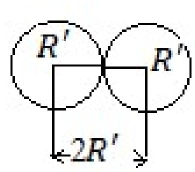
$$
\begin{aligned}
E_0 &= (E_s + E_c)_0 \\
&= a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} \\
&= 4\pi {r_0}^2 S A^{2/3} + \frac{3}{5}\frac{Z^2 e^2}{4\pi\epsilon_0 r_0 A^{1/3}}
\end{aligned}
$$

In the fission process, E_s tend to restore the original shape, while E_c have the effect of increasing the deformation, because the surface energy is minimum for the sphere while the Coulomb energy decreases with increased deformation.



(1) For lighter nuclei: $E_c < E_s$

$R' = r_0(A/2)^{1/3}$, the radius of each fragment (spherical)



So, the energy just after separation is

$$
\begin{aligned}
E &= 2\left[a_2 (A/2)^{2/3} + a_3 \frac{(Z/2)^2}{(A/2)^{1/3}}\right] + \frac{(Ze/2)^2}{4\pi\epsilon_0 \times 2R'} \\
&= 2\left[4\pi {r_0}^2 S (A/2)^{2/3} + \frac{3}{5}\frac{(Ze/2)^2}{4\pi\epsilon_0 r_0 (A/2)^{1/3}}\right] + \frac{(Ze/2)^2}{4\pi\epsilon_0 \times 2r_0 (A/2)^{1/3}}
\end{aligned}
$$

Hence, the critical energy of deformation to cause fission is (neglecting the energy of the neck)

$$\Delta E_{cr} = E - E_0$$

$$y = 0.26 - 0.215\,x$$

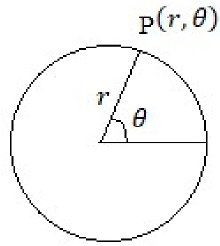
Where,

$$y = \frac{\Delta E_{cr}}{4\pi {r_0}^2 A^{2/3} S} \qquad \& \qquad x = \frac{(Z^2/A).3e^2}{4\pi\epsilon_0 \times 40\pi\, {r_0}^3\, S}$$

So, the critical energy of deformation for causing fission is a linear function of the parameter of Z^2/A for light nuclei.

(2) For very heavy nuclei: $E_c > E_s$

Hence even a slight initial deformation of the drop will tend to build up against the forces of surface tension.



Assuming the deformation to have an axial symmetry, we can write the distance $r(\theta)$ of a point P on the nuclear surface at the polar angle θ to be

$$r(\theta) = R + R.\sum_{l=0}^{\infty} \alpha_l P_l(\cos\theta)$$
$$= R[1 + \alpha_2 P_2(\cos\theta) + \alpha_3 P_3(\cos\theta) + \alpha_4 P_4(\cos\theta) + \cdots]$$

$R =$ the radius of the original undeformed spherical nuclear droplet.

Now the S.E. for the **undeformed** drop is

$$E_{SO} = 4\pi R^2 S = 4\pi {r_0}^2 S A^{2/3}$$

And the S.E. for the **deformed** drop is

$$E_S = 4\pi\, r^2(\theta) S$$
$$= 4\pi.R^2[1 + \alpha_2 P_2(\cos\theta) + \alpha_3 P_3(\cos\theta) + \alpha_4 P_4(\cos\theta) + \cdots]^2.S$$
$$= 4\pi {r_0}^2 S A^{2/3}\left[1 + \alpha_2\left(\frac{3}{2}cos^2\theta - \frac{1}{2}\right) + \cdots\right]^2$$
$$= 4\pi {r_0}^2 S A^{2/3}\left[1 + \frac{2}{5}{\alpha_2}^2 + \frac{5}{7}{\alpha_3}^2 + \cdots\right]$$

Therefore,

$$\Delta E_S = E_S - E_{S0}$$
$$= 4\pi {r_0}^2 S A^{2/3}\left[\frac{2}{5}{\alpha_2}^2 + \frac{5}{7}{\alpha_3}^2 + \cdots\right]$$
$$= E_{S0}\left[\frac{2}{5}{\alpha_2}^2 + \frac{5}{7}{\alpha_3}^2 + \cdots\right]$$

Now the Coulomb energy for the **undeformed** drop is

$$E_{CO} = \frac{3}{5}\frac{Z^2e^2}{4\pi\epsilon_0 R} = \frac{3}{5}\frac{Z^2e^2}{4\pi\epsilon_0 r_0 A^{1/3}}$$

And the Coulomb energy for the **deformed** drop is

$$E_C = \frac{3}{5}\frac{Z^2e^2}{4\pi\epsilon_0 r_0 A^{1/3}}\left[1 + \alpha_2\left(\frac{3}{2}cos^2\theta - \frac{1}{2}\right) + \cdots\right]^{-1}$$

Therefore,

$$\Delta E_C = E_C - E_{C0}$$
$$= \frac{3}{5}\frac{Z^2e^2}{4\pi\epsilon_0 r_0 A^{1/3}}\left[-\frac{1}{5}{\alpha_2}^2 - \frac{10}{49}{\alpha_3}^2 - \cdots\right]$$

$$= -E_{CO}\left[\frac{1}{5}\alpha_2{}^2 + \frac{10}{49}\alpha_3{}^2 + \cdots\right]$$

Since, α's are very small numbers, hence we can neglect the higher order of α values.
We get,

$$\Delta E = \Delta E_C + \Delta E_S = \frac{1}{5}\alpha_2{}^2(2E_{SO} - E_{CO})$$
$$= \frac{\alpha_2{}^2}{5}\left(2a_2A^{2/3} - a_3\frac{Z^2}{A^{1/3}}\right)$$
$$< 0, \qquad for\ Z\ large$$

If $\Delta E > 0,\ i.e.,\ \ if \quad 2E_{SO} > E_{CO}$, the drop is stable to small distortions.
If $\Delta E < 0,\ i.e.,\ \ if \quad 2E_{SO} < E_{CO}$, the fission may occur spontaneously.
So, for fission, we have

$$2E_{SO} < E_{CO}$$
$$or,\quad 2\times 4\pi r_0{}^2SA^{2/3} < \frac{3}{5}\frac{Z^2e^2}{4\pi\epsilon_0 r_0A^{1/3}}$$
$$or,\quad 2\times a_2A^{2/3} < a_3\frac{Z^2}{A^{1/3}}$$
$$or,\quad \frac{Z^2}{A} > \frac{2a_2}{a_3}$$

Since, $\quad a_2 = 0.019114\ u\ \&\ a_3 = 0.0007626\ u$

$$So,\quad \frac{Z^2}{A} > 50$$

So, the limiting value of $\frac{Z^2}{A}$ is

$$\left(\frac{Z^2}{A}\right)_{lim} \approx 50$$

The ratio $\frac{E_{CO}}{2E_{SO}}$ is known as the **critical parameter**.
Therefore, for occurring the instantaneous spontaneous fission

$$\frac{Z^2}{A} > \left(\frac{Z^2}{A}\right)_{lim}$$

This sets the limit for a nucleus to be stable against surface force.
For example, ${}^{238}_{92}U$, we have $\frac{Z^2}{A} \approx 35.56 < 50$. That is the nucleus ${}^{238}_{92}U$ is stable against instantaneous spontaneous fission.
Bohr and Wheeler have calculated the critical deformation for which the deformation energy ΔE is maximum.
In this case

$$\Delta E_{cr} = 4\pi r_0{}^2SA^{2/3}\left[\frac{98}{135}(1-x)^3 + \frac{11368}{34425}(1-x)^4 + \quad \dots\right]$$
$$y(x) = \frac{98}{135}(1-x)^3 + \frac{11368}{34425}(1-x)^4 + \quad \dots$$

Where,

$$y(x) = \frac{\Delta E_{cr}}{4\pi r_0{}^2SA^{2/3}} \qquad \& \qquad x = \frac{Z^2}{A}\Big/\left(\frac{Z^2}{A}\right)_{lim}$$

The theory discussed above is classical.

References:

1. S. N. Ghoshal (2015). Nuclear Physics. S. Chand & Company Pvt. Ltd., New Delhi.
2. D. C. Tayal (2015). Nuclear Physics. Himalaya Publishing House Pvt. Ltd., Mumbai.