# Ferrimagnetism

In many ferromagnetic crystals the saturation magnetization at

T = 0 K does not correspond to parallel alignment of the magnetic moments of the constituent paramagnetic ions, even in crystals for which the individual para- magnetic ions have their normal magnetic moments.

The most familiar example is magnetite,

$Fe_3O_4$ or $FeO \cdot Fe_2O_3$

.

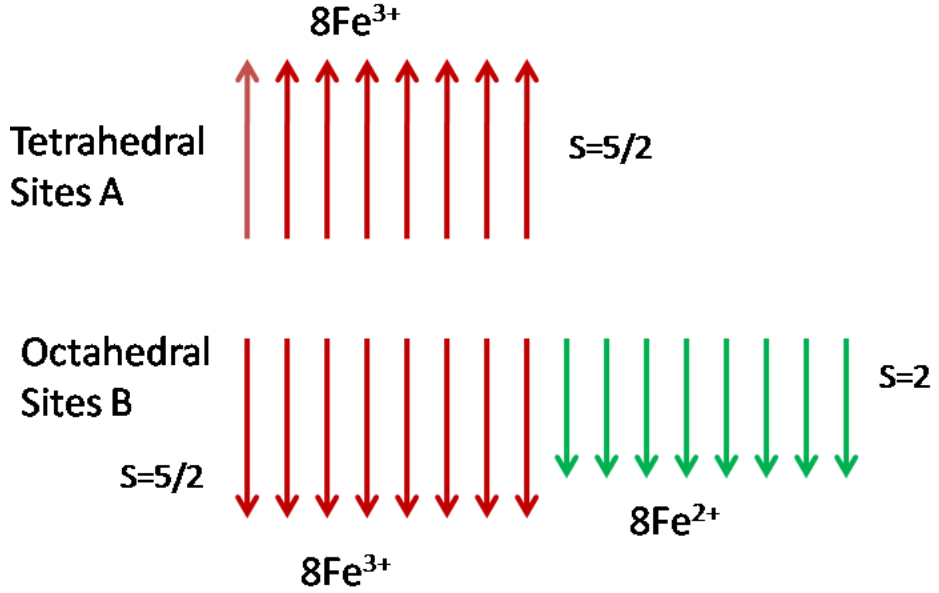


Fig.1

**Spin arrangements in magnetite**

Ferric ($Fe^{3+}$) ions are in a state with spin S =5/2 and zero orbital moment. Thus each ion should contribute $5\mu_B$ to the saturation moment. The ferrous ($Fe^{2+}$) ions have a spin of 2 and should contribute $4\mu_B$ , apart from any residual orbital moment contribution. Thus the effective number of Bohr magnetons per $Fe_30_4$ formula unit should be about 2 X 5 + 4 = 14 if all spins were parallel

But the observed value of Bohr magnetons per $Fe_3O_4$ formula unit is 4.1. The discrepancy is accounted for if the moments of the $Fe^{3+}$ ions are antiparallel to each other: then the observed moment arises only from the $Fe^{2+}$ ion, as in Fig. 1. Neutron diffraction results agree with this model.

A systematic discussion of the consequences of this type of spin order was given by L. Néel with reference to an important class of magnetic oxides known as **ferrites**. The usual chemical formula of a ferrite is $\mathbf{MO \cdot Fe_2O_3}$, where M is a divalent cation, often Zn, Cd, Fe, Ni, Cu, Co, or Mg.

The term **ferrimagnetic** was coined originally to describe the ferrite-type ferromagnetic spin order such as Fig. 1, and by extension the term covers almost any compound in which some ions have a moment antiparallel to other ions. Many fcrrimagnets are poor conductors of electricity, a quality exploited in applications such as rf transformer cores.

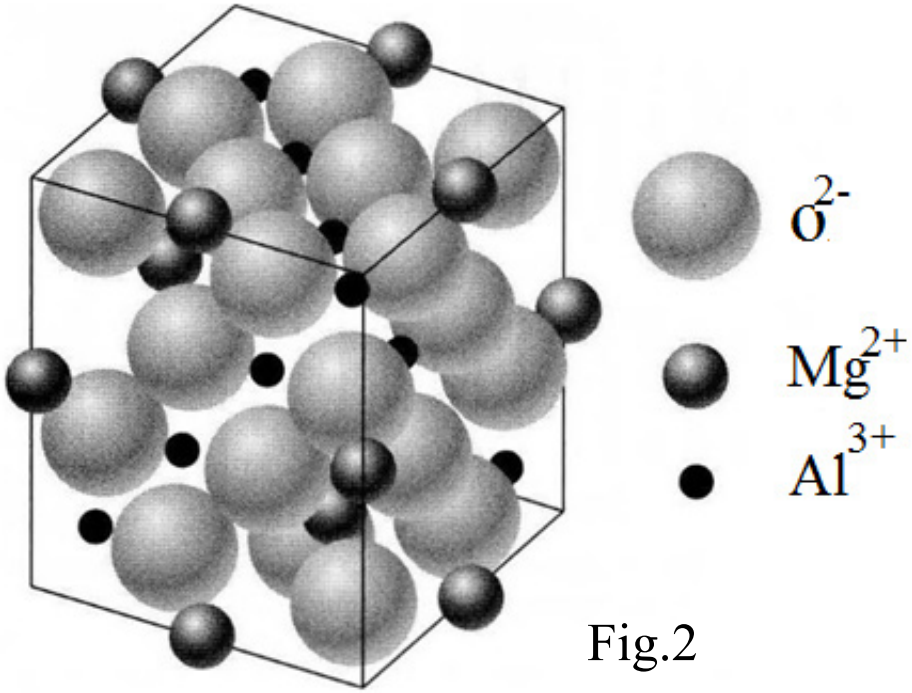


Fig.2

Crystal structure of the mineral **spinel** $MgAl_2O_4$; the $Mg^{+2}$ ions occupy tetrahedral sites, each surrounded by four oxygen ions; the $Al^{+3}$ occupy octahedral sites, each surrounded by six oxygen ions.

The cubic ferrites have the **spinel** crystal structure shown in Fig. 2. There are eight occupied tetrahedral (or *A)* sites and 16 occupied octahedral(or *B)* sites in a unit cube. The lattice constant is about 8 Å.

A remarkable feature of the spinels is that all exchange integrals $J_{AA,}\, J_{AB}$ and $J_{BB}$ are negative and favor *antiparallel* alignment of the spins connected by the interaction

. But the *AB* interaction is the strongest, so that the *A* spins are parallel to each other and the *B* spins are parallel to each other, just in order that the *A* spins may be antiparallel to the *B* spins. If *J* in $U = -2J\mathbf{S}_i \cdot \mathbf{Sj}$ is positive, we say that the exchange integral is ferromagnetic; if *J* is negative, the exchange integral is antiferromagnetic.

We now prove that three antiferromagnetic interactions can result in ferrimagnetism. The mean exchange fields acting on the *A* and *B* spin lattices may be written

$$\mathbf{B}_A = -\lambda \mathbf{M}_A - \mu \mathbf{M}_B \qquad \mathbf{B}_B = -\mu \mathbf{M}_A - \nu \mathbf{M}_B \qquad 1$$

taking all mean field constants $\lambda, \mu, \nu$ to be positive. The minus sign then corresponds to an antiparallel interaction. The interaction energy density is

$$U = -\tfrac{1}{2}(\mathbf{B}_A \cdot \mathbf{M}_A + \mathbf{B}_B \cdot \mathbf{M}_B) = \tfrac{1}{2}\lambda \mathbf{M}_A^2 + \mu \mathbf{M}_A \cdot \mathbf{M}_B + \tfrac{1}{2}\nu \mathbf{M}_B^2 \qquad 2$$

Interaction energy is lower when $\mathbf{M_A}$ is antiparallel to $\mathbf{M_B}$ than when $\mathbf{M_A}$ is parallel to $\mathbf{M}_{\mathbf{B}}$. The energy when antiparallel should be compared with zero, because a possible solution is $M_A = M_B = 0$. Thus when

$$\mu M_A M_B > \tfrac{1}{2}(\lambda M_A^2 + \nu M_B^2) \qquad 3$$

The ground state will have $M_A$ directed oppositely to $M_B$

***Curie Temperature and Susceptibility of Ferrimagnets***

We define separate Curie constants $C_A$ and $C_B$ for the ions on the *A* and *B* sites. For simplicity, let all interactions be zero except for an antiparallel interaction between the *A* and *B* sites:

$B_A = -\mu M_B$, $\qquad B_B = -\mu M_A$

, where μ is positive. We have in the mean field approximation

$$M_A T = C_A(B_a - \mu M_B) \qquad M_B T = C_B(B_a - \mu M_A) \qquad 4$$

where $B_a$ is the applied field.

These equations have a nonzero solution *for* $M_A$ and $M_B$ in zero applied field if

$$\begin{vmatrix} T & \mu C_A \\ \mu C_B & T \end{vmatrix} = 0 \qquad 5$$

so that the ferrimagnetic Curie temperature is given by

$$T_c = \mu (C_A C_B)^{1/2}$$

We solve (35) *for* $M_A$ and $M_B$ to obtain the susceptibility at $T > Tc$

$$\chi = \frac{M_A + M_B}{B_a} = \frac{(C_A + C_B)T - 2\mu C_A C_B}{T^2 - T_c^2} \qquad 6$$

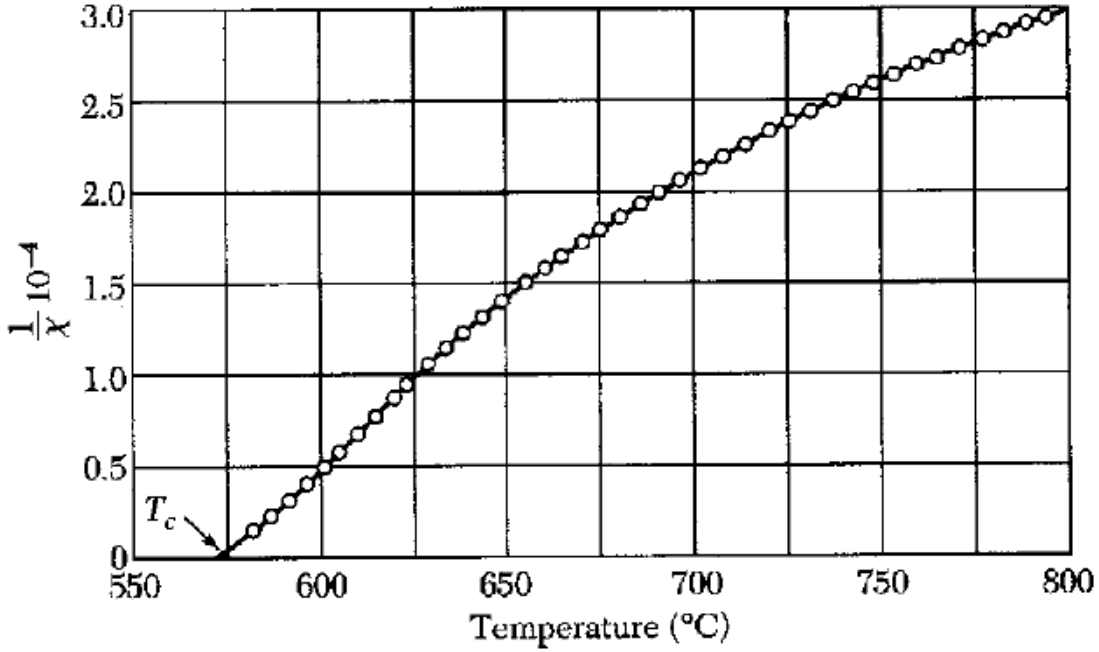


Fig.3 Reciprocal susceptibility of magnetite, $FeO \cdot Fe_2O_3$

Experimental values for $Fe_3O_4$ are plotted in Fig.3
The curvature of the plot of $1/\chi$ versus $T$ is a characteristic feature of a ferrimagnet.

**Ferrites**

have magnetic properties which resemble in certain respect s to those metallic ferromagnetism For example they show hysteresis, spontaneous magnetization, and associated phenomena.

They are technically important solids: their technical importance are due to the fact

(i) they are not metals but ionic crystal of resistivity in the range $10^2$ to $10^6$ ohm-cm This puts them in insulator or semiconductor class if compared with $10^{-6}$ ohm –cm for iron. This means that power losses due to eddy currents are much reduced in these solids and are therefore well suited for microwave frequency applications

(ii) Their magnetization can be controlled by changing the relative percentage of different constituent ions.

# Iron Garnets.

The iron garnets are cubic ferrimagnetic insulators with the general formula $M_3Fe_5O_{12}$, where M is a trivalent metal ion and the Fe is the trivalent ferric ion (S = 5/2, $L$ = 0). An example is yttrium iron garnet $Y_3Fe_5O_{12}$,known as YIG. Here $Y^{3+}$ is diamagnetic.

The net magnetization of YIG is due to the resultant of two oppositely magnetized lattices of $Fe^{3+}$ ions. At absolute zero each ferric ion contributes $\pm 5\mu_B$ to the magnetization, but in each formula unit the three $Fe^{3+}$ ions on sites denoted as *d* sites are magnetized in one sense and the two $Fe^{3+}$ ions on *a* sites are magnetized in the opposite sense, giving a resultant of $5\mu_B$ per formula unit in good agreement with the measurements of Geller *et al.*

The mean field at an *a* site due to the ions on the *d* sites is $B_a = -(1.5 \times l0^4) M_d$. The observed Curie temperature 5.59 K of YIG is due to the *a-d* interaction. The only magnetic ions in YIG are the ferric ions. Because these are in an $L = 0$ state with a spherical charge distribution, their interaction with lattice deformations and phonons is weak. As a result YIG is characterized by very narrow linewidths in ferromagnetic resonance experiments.