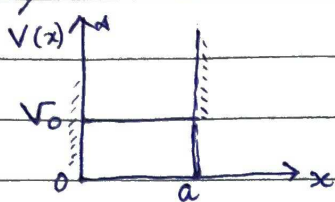
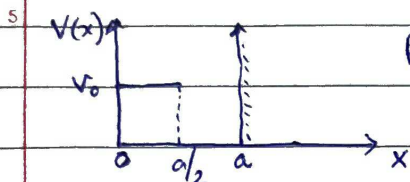


1) We all know the wave function for infinite square well potential is $\Psi_n^0(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$. The perturbation is shown in figure.

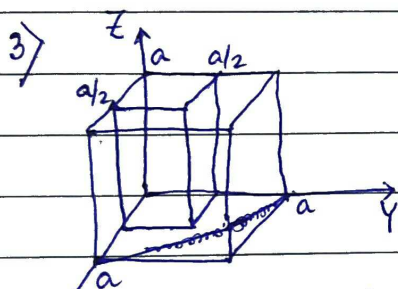


Find the first order correction to energy.



For this case calculate the first order correction to energy.

2) Let there is a bump in a infinite square well potential $H' = A\delta(x - \frac{a}{2})$ where A is constant. Find out the first order correction to energy and wave function.



The perturbation in a 3-dimensional infinite square well is $H' = \begin{cases} V_0, & \text{for } 0 < x < a/2 \text{ and } 0 < y < a/2 \\ 0, & \text{otherwise} \end{cases}$

Calculate the correction of energy and eigenfunction.

4) The perturbation of 3-D harmonic oscillator is $H' = \lambda x^2 y z$, $\lambda = \text{constant}$. Calculate correction to ground state and 1st excited state of energy.

5) Consider a quantum system with three linearly independent states. Let the Hamiltonian is $H = V_0 \begin{pmatrix} 1-\epsilon & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & \epsilon & 2 \end{pmatrix}$ where $V_0 = \text{constant}$ and $\epsilon = \text{small number}$.

(i) Write down the eigenvalue and eigenvector of unperturbed Hamiltonian.

(ii) Solve the exact eigenvalues of H. Expand each of them as power series of ϵ , upto second order.

(iii) Use first and second order nondegenerate perturbation theory to find out the eigenvalues for the state that grows out of nondegenerate eigenvalue of H^0 .

(iv) Use degenerate perturbation theory to find the first order correction to the two initially degenerate eigenvalues and compare the results.

6) Calculate some useful expectation value

$$\langle \frac{1}{r} \rangle = \frac{1}{n^2 a_0} ; \langle \frac{1}{r^2} \rangle = \frac{1}{(l+1/2)n^3 a^2} ; \langle \frac{1}{r^3} \rangle = \frac{1}{l(l+1/2)(l+1)n^3 a^3}$$