Equations of Romer's Model

$$Y = L_Y^{1-\alpha}(x_1^{\alpha} + x_2^{\alpha} + \cdots + x_A^{\alpha})$$

or $$Y = L_Y^{1-\alpha}\sum_{i=1}^{A} x_i^{\alpha} \quad (1)$$

$$\dot{A} = \theta L_A^{\lambda} A^{\phi} \quad (2)$$

$$L = L_A + L_Y \quad (3)$$

$$L_A = s_A L \quad (4)$$

$$\frac{\dot{L}}{L} = n \quad (5)$$

$$K = \sum_{i=1}^{A} x_i \quad (6)$$

$$\dot{K} = s_K Y - \delta K \quad (7)$$

$$x_i = \bar{x}, \quad i = 1, 2, \ldots, A \quad (8)$$

$$Y = A L_Y^{1-\alpha}\bar{x}^{\alpha} \quad (9)$$

$$K = A\bar{x} \Rightarrow \bar{x} = \frac{K}{A} \quad (10)$$

$$Y = A L_Y^{1-\alpha}\left(\frac{K}{A}\right)^{\alpha} = (A L_Y)^{1-\alpha} K^{\alpha} \quad (11)$$

$$Y = (A s_Y L)^{1-\alpha} K^{\alpha} \quad (12)$$

where $$s_Y = 1 - s_A \quad (13)$$

$$\frac{\dot{Y}}{Y} = (1-\alpha)\left(\frac{\dot{A}}{A} + \frac{\dot{s}_Y}{s_Y} + \frac{\dot{L}}{L}\right) + \alpha\frac{\dot{K}}{K} \quad (14)$$

$$\left(\frac{\dot{Y}}{Y}\right)^{*} = (1-\alpha)\left(\frac{\dot{A}}{A} + \frac{\dot{s}_Y}{s_Y} + \frac{\dot{L}}{L}\right) + \alpha\left(\frac{\dot{Y}}{Y}\right)^{*} \quad (15)$$

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$$\left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}\right)^{*} = \frac{\dot{A}}{A} \quad \text{—— (16)}$$

$$\frac{\dot{A}}{A} = \vartheta (S_A L)^{\lambda} A^{\phi-1} \quad \text{—— (17)}$$

$$\lambda\left(\frac{\dot{S}_A}{S_A} + \frac{\dot{L}}{L}\right) - (1-\phi)\frac{\dot{A}}{A} = 0 \quad \text{—— (18)}$$

$$\left(\frac{\dot{A}}{A}\right)^{*} = \frac{\lambda n}{1-\phi} \quad \text{—— (19)}$$

$$\left(\frac{Y}{L_Y}\right) = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A \quad \text{—— (20)}$$

$$\left(\frac{Y}{L}\right) = (1-S_A)\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A \quad \text{—— (21)}$$

$$\left(\frac{K}{Y}\right)^{*} = \frac{S_K}{n + \frac{\lambda n}{1-\phi} + \delta} \quad \text{—— (22)}$$

$$\frac{\dot{A}}{A} = \vartheta (S_A L)^{\lambda} A^{\phi-1} = \frac{\lambda n}{1-\phi} \quad \text{—— (23)}$$

$$A^{*} = \left(\frac{\vartheta(1-\phi)}{\lambda n}\right)^{\frac{1}{1-\phi}} (S_A L)^{\frac{\lambda}{1-\phi}} \quad \text{—— (24)}$$

$$\left(\frac{Y}{L}\right)^{*} = (1-S_A)\left(\frac{S_K}{n + \frac{\lambda n}{1-\phi} + \delta}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\vartheta(1-\phi)}{\lambda n}\right)^{\frac{1}{1-\phi}} \cdot \cancel{S_A} (S_A L)^{\frac{\lambda}{1-\phi}} \quad \text{—— (25)}$$

$$g_A = \frac{\dot{A}}{A} = \vartheta (S_A L)^{\lambda} A^{\phi-1} \quad \text{—— (26)}$$

$$\frac{\dot{g}_A}{g_A} = \lambda\left(\frac{\dot{S}_A}{S_A} + n\right) - (1-\phi) g_A \quad \text{—— (27)}$$

$$g_A > \frac{\lambda n}{1-\phi} + \frac{\lambda}{1-\phi}\frac{\dot{S}_A}{S_A} \quad \text{—— (28)}$$

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