M.Sc. Course
in
Applied Mathematics with Oceanology
and
Computer Programming

Semester-IV

Paper-MTM402 Unit-1

Fuzzy Mathematics with Applications

(Introduction to Fuzzy Sets)

Unit Structure:

1.1 Introduction

The classical sets divide the world into two distinct classes viz. white and black, true and false. As for example in a collection "all integers" if we consider a sub-collection of all "even integers" and ask whether an integer belongs to that sub-collection or not, the answer will be either yes or no. No ambiguity remains in the answer. But in the same collection of all integers if we consider a sub-collection of all "large integers" and ask whether an integer, say 2500790, belongs to the sub-collection or not, then ambiguity comes in the answer. In some situation the answer will be yes (e.g. in the study of number of students in different classes in a school), in some situation the answer will be no (e.g. in the study of number of atoms taking part in a chemical reaction), again in some situation ambiguity occurs in the answer, the answer may be neither yes nor no. Let us consider another example. In the collection of "all boys" if we consider the sub-collection "good boys" and ask whether a particular boy is a member of this sub-collection or not, the answers will be different from different persons. His friend or his mother will answer 'yes', his enemy will answer 'no', whereas a common people will answer "I don't no". Someone may say that the boy is neither good nor bad.

Thus in our natural language, there is a great deal of imprecision,vagueness or fuzziness. The following are some more examples.

(i) The classification of certain objects as "small".

(ii) The description of a human characteristic such as "healthy" or as "tall".

(iii) The classification of people by age such as "old".

(iv) The classification of patients as "depressed".

(v) The classification of flowers as "red".

(vi) The classification of students as "intelligent".

In the above examples it may be impossible to decide whether an individual object belongs to the subset or not. There is no sharp boundary between members and non members and hence the concept of gradation of membership, or degree of membership becomes necessary. To discuss the situation of partial membership, let us consider the following example. Let us consider the universal set as "all students attending the inaugural ceremony held in a room of a school". Let the students be listening the function in that packed up room both in sitting and standing position and the standing students are standing both inside and outside of that specified room. Let us consider the subset "the students remaining inside of the room".

Here the student remaining completely inside of the room has full membership having membership grade one and the student remaining completely outside of the room has no membership having membership grade zero. Now question arises "what about the membership grade of a student standing at the door whose some part of the body is inside and some part outside of the room?" Naturally, this student will have a partial membership, and the grade of membership will be some number in between zero and one. The value of membership grade depends on the percentage of his body remaining inside of the room. If he has 50% of his body inside then the grade of membership is $\frac{1}{2}$, whereas if he has 75% of his body inside then the grade of membership is $\frac{3}{4}$. In general if he has $x\%$ his body inside then the grade of membership is $\frac{x}{100}$.

These situations where multigrade of membership is needed, "fuzzy set" is the tool. Fuzzy sets deals with objects that are "matter of degree" with all possible grades of truth between yes and no, and the shades of grey between white and black.

Let us consider one more example. If someone ask the question "Is Ram a student?" The answer is definite, Yes or No. This is a crisp situation. But if the question is "Is Ram honest?" The answer here is not definite. A variety of answers will come as "yes honest" or "extremely honest" or "extremely dishonest" or "honest at times" or "very honest" or "No" etc. This situation is fuzzy.

1.2 The Birth of Fuzzy Set Theory

In July,1964 Zadeh was in New York city visiting his parents. He was then invited by Richard Belman to spend part of the summer at Rand Corp to work on problems in "pattern classification" and "system analysis". With this upcoming work on his mind, his thoughts often turned to the use of "imprecise categories for classification".

One night in New York, Zadeh had a dinner engagement with some friends. But it was canceled, and he spent the evening alone in his parents apartment, and the idea of grade of membership, which is the backbone of fuzzy set theory, occurred in his mind. This important event gave the birth of fuzzy logic technology and fuzzy set theory with the publication of his seminal paper on fuzzy sets in 1965.

The concept of fuzzy sets had to encounter sharp and strong criticism from academic community. Some rejected it because of the name, without knowing the content in detail. Others rejected it because of the theory's emphasis on imprecision. The funding agency of Zadeh "National Science Foundation" even was suggested by Congress as "Not to waste Government Funds".

1.3 Transition from traditional view to modern view

A paradigmatic change in science occurred with the concept of uncertainty. In science, this change occurred as a gradual transition. The traditional view insisted that uncertainty is undesirable in science and should be avoided by all possible means. According to the traditional view, science should strive for certainty in all its manifestations and so science should deal with only precision, specificity, sharpness, consistency etc. Accordingly, uncertain situations like imprecision, nonspecificity, vagueness, inconsistency etc. should be avoided as they are regarded unscientific.

The transition from the traditional view to the modem view of uncertainty began in the 19th century when study of molecular level became essential in physics. The need for fundamentally different approach to the study of physical processes at the molecular level motivated the development of relevant statistical methods viz statistical mechanics. The role-played in Newtonian mechanics by the calculus, which involves no uncertainty, is replaced in statistical mechanics by probability theory. The analytic methods and statistical methods are highly complementary. The analytic methods based upon the calculus are applicable only to problems involving a very small number of variables that are related to one another in a predictable way. The statistical method on the other hand has exactly opposite characteristic as they require a very large number of variables which are related to one another in a very high unpredictable manner.

The purpose of probability theory is to capture uncertainty of a particular type known as random uncertainty. But there are many uncertainties which are not of random type. They are called non-random uncertainties and are associated with vagueness, with imprecision and with lack of intonation regarding a particular element of the problem at hand. Fuzzy set theory is a marvelous tool for handling these non-random uncertainties. The underlying power of fuzzy set theory is that it can use linguistic variables rather than quantitative variables, to represent imprecise concepts.

1.4 Concept of uncertainty

Uncertainty arise from the following:
i) complexity ii) ignorance iii) chance iv) randomness v) imprecision vi) inability to perform adequate measurements vii) inconsistency viii) vagueness from natural language etc.

Uncertainties can be divided into following two categories (i) Random uncertainty and (ii) Non-random uncertainty.

Random uncertainty occurs due to rack of information, the future state of the system is not known completely. It describes uncertainty in the occurrence of the event. This type of uncertainty is handled by probability theory.

Non random uncertainty occurs due to vagueness concerning the description of the semantic meaning of the events, phenomena or statements. It describes the ambiguity of an event. This type of uncertainty is handled by fuzzy set theory.

Only a small portion of the "information world" is certain, a vast portion of the information world is actually uncertain. Again, in the uncertain information world the portion of the non-random type uncertainty is much larger than the portion of the random type uncertainty. The pi-diagram given in Fig. 1.1 shows these proportions.

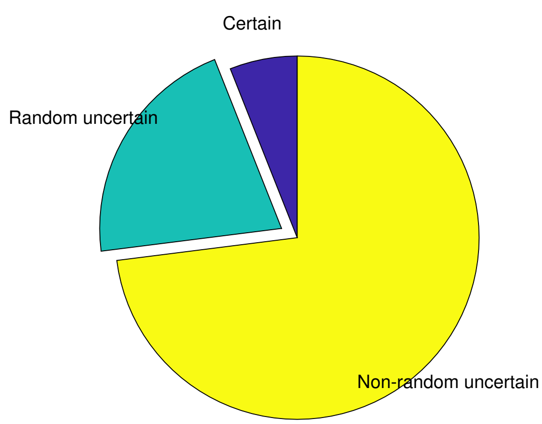

Figure 1.1: Information World (real situation)

1.5 Random uncertainty verses Fuzzy uncertainty

The classical concept of set holds for both the deterministic and the stochastic cases. The random uncertainty occurs if the future state of the system is not known. It is handled by probability theory. Stationary random processes are those that arise out of chance, where the chances represent frequencies of occurence that can be measured. Problems like drawing balls from an urn, tossing coin and dice, drawing cards from a pack are examples of stationary random processes.

Now we see how to recognize the random behavior of uncertainties? For example, are the following uncertainties random?

(i) uncertainty in the weather prediction

(ii) uncertainty in choosing clothes for the next day

(iii) uncertainty in buying a car

(iv) uncertainty in your preference in colors

(v) uncertainty your ability in parking a car

(vi) uncertainty in causing cancer for consuming tabaco

Although it is possible to model all of these forms of uncertainty with various classes of random processes, the solution obtained may not be reliable. Treatment of these forms of uncertainty using fuzzy logic should also be done with caution. We should study the character of the uncertainty first, then we are to choose an appropriate approach to develop a model of the process. Again same problem may have many features. As for example, let the weather report suggests that "there is a 80% chance of rain tomorrow". It may mean that there has been rain on tomorrow's date for 80 of the last 100 years. It may mean that somewhere in your community 80% of the land area will receive rain. Again it may mean that 80% of the time of tomorrow it will be raining. Also humans often deal with these forms of uncertainty linguistically such as "it will likely rain tomorrow". With this crude assessment of the possibility of rain, humans can still make appropriately accurate decisions about the weather.

Another important point is to be noted here. The statement "I think it will rain today" is not certain. This statement may be true with a degree of certainty. Let the level of certainty be 0.8. It is the truth value of the statement. The degree of certainty sounds like probability. But it is not quite the Same. Probabilities for mutually exclusive events cannot add up to more than one, but their fuzzy values may. Suppose that probability of a cup of tea being hot is 0.8 and so probability of being cold is 0.2. The probabilities must add up to 1. On the other hand, the truth value of the proposition "a cup of tea is hot" may be 0.8 and the truth value of the proposition "a cup of tea is cold" may be 0.3. The sum of these two truth values here is 1.1 not 1.

The problems occurring in the real world are in general complex owing to an element of uncertainty either in the parameters which define the problem or in the situation in which the problem occurs. Probability theory can be applied only to a situation whose characteristics are based on random process i.e. process in which the occurrence of events is strictly determined by chance. In reality there are a large class of problems whose uncertainty is characterized by a non-random process. Here the uncertainty may arise due to following reasons:

(i) due to partial information about the problem

(ii) due to information which is not fully reliable

(iii) due to inherent imprecision in the language

(iv) due to receipt of information from more than one source which are conflicting.

Fuzzy set theory has immense potential for effective solving of uncertainty of above types which arc 'non-random in nature.

We should not be confused between probability value and membership value. If we ask the question of "what is the probability of an individual x to be a member of a subset A?". The answer may be "the probability for x to be a member of A is 90%". Here the chance of the correct prediction for membership of x is 90% membership in the set A and 10% non-membership in the same set. we may note that in the classical set theory, it is not permissible for an individual to be a partial member of a set. Partial membership is only permissible in fuzzy set theory.

Let us consider an example and see how we can combine two types of uncertainties. Let a bag contains ten identical red balls with different gradation of red colour. Let the grades of 10 balls be 0.95, 0.93, 0.91, 0.9, 0.9, 0.7, 0.7, 0.0, 0.0, 0.0. Here the balls are identical and three balls have membership grade 0.0 i.e. three balls are completely non-red. The other seven balls are red having different gradation in red colour. If a ball is drawn from the bag at random then the probability that the ball drawn is red is given by $\frac{7}{10}$ i.e. 0.7 as in the bag there are 7 red balls and 3 non-red balls. Here the drawn ball may be anyone of the 10 balls, even non-red one. If the experiment is performed a large number of times then we expect 70% of the drawn balls to be red.

On the other hand "Grade of membership of a ball is 0.7" means a particular ball whose grade of redness is 0.7, it can never be any other ball having grade different from O.7, however it may be any one of the two ball having grade 0.7.

To be more precise in the experiment of drawing red balls at random from the bag we note that all balls are not red of same grade i.e. they have diffrent gradetion of being red. So the statement that there are 7 red balls out of 10 balls is not fully correct as someone may disagree to regard balls with gradetion $O.7$ as red. We give the following argument to tackle the situation. First, we have to select a value of membership, above which we would be willing to regard the colour as red. For example, any ball with a membership value above 0.8 in the fuzzy set of "red balls" would be considered as red. Secondly we would have then to know the proportion of the balls in the bag that have membership values above 0.8. The number of such balls is 5 having membership grade 0.95, 0.93, 0.91, 0.9 & 0.9.

Thus the probability of randomly selecting red balls from be bag is $\frac{5}{10} = \frac{1}{2}$. On the other hand if we regard the balls having grade more than 0.9 as red then the probability is $\frac{3}{10}$.

Hence first we have to access the ambiguity of redness and then we are to determine the probability. Thus we have been able to combine both types of uncertainties random and non-random .

Finally we again recollect that

(i) Random uncertainty describes uncertainty in the occurrence of the event i.e. Uncertainty arising due to random occurrence which is handled by probability theory.

(ii) Non-random uncertainty describes ambiguity of an event. It arises due to belongingness which is handled by set theory .

1.6 Power of humanity thinking

Our common way to convey information is living language. By the very nature of living language, it is vague and imprecise. Yet it is most powerful form of communication and information exchange among humans.

Despite the vagueness, humans have very little trouble in understanding one another's concept and ideas. Our understaning is based largely on imprecise human reasoning. This imprecision is a form of information that can be quite useful to humans.

Human thinking and feeling, in which ideas, pictures, images and value systems are formed, has certainly more concepts or comprehension than our daily language has words. Our thinking is unlimited but words in a dictionary is definitely limited.

The diagram given in Fig. 1.2 shows the real situations and our power of thinking.

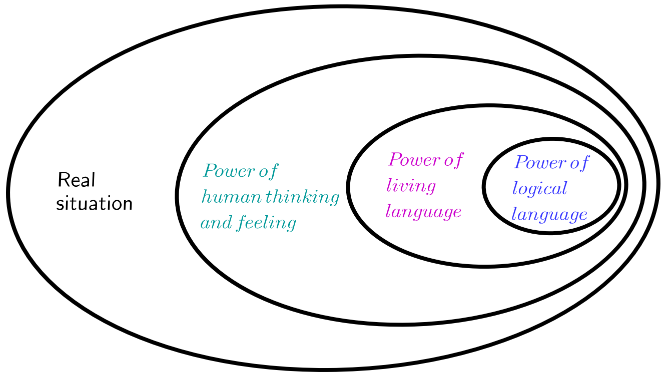

Figure 1.2: Power of humanity thinking

1.7 Applications of fuzzy set theory

Fuzzy set occurs almost in all areas in which human judgement, evaluations and decisions are important. These are the areas of decision-making, reasoning, learning and so on.

More specifically application area of fuzzy set theory covers

(i) Engineering

(ii) Phycology

(iii) Medicine

(iv) Ecology

(v) Artificial Intelligence

(vi) Decision theory

(vii) Pattern recognition

(viii) Sociology

(ix) Meteorology

(x) Computer science

(xi) Manufacturing and so on.

Practical implementation on fuzzy set theory are as follows :

(i) Fuzzy air conditioner that controls temperature changes according to human comfort.

(ii) Fuzzy washing machine which detect the colour and the kind of cloth present in the machine and accordingly acts i.e. controls the revolution and select the type and amount of detergent.

(iii) Fuzzy videography offering fuzzy focussing and image stabilization.

(iv) Fuzzy computer which controls a number of stations in the subway system, the ride is so smooth that the riders do not need to hold anything.

(v) Fuzzy anti-skid braking system to luxary cars.

(vi) Fuzzy rice cookers.

(vii) Fuzzy vacuum cleaners.
and so on.

important Quotations:
Relating to the notion of fuzzy set theory and fuzzy logic great thinkers and philosophers made remarkable statements. We state here some of them.

(i) **Charles Sanders Pierce (1839-1914):** He laughed at the 'sheep and goat separators' who split the world into true and false. "All that exists is continuous and such continuums govern knowledge".

(ii) **Bertrand Russell (1872-1970):** "Both vagueness and precision are features of language, not reality. Vagueness clearly is a matter of degree". All traditional logic assumes precise symbols. So traditional logic is not applicable to this terrestrial life.

(iii) **Jan Lukasiewicz (1878-1956) :** He proposed a formal model of vagueness, a logic 'based on more values than TRUE or FALSE'. 1 stands for TRUE, 0 stands for FALSE and $\frac{1}{2}$ stands for possible. (Actually the trhee-valued logic by Lukasiewicz stayed just one step away from the multivatued fuzzy logic by Zadeh and can be considered as its closest relative)

(iv) **Max Black (190989):** He proposed a degree as a measure of vagueness.

(v) **AIbert Einstein (1879-1955) :** "So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality".

(vi) **Lotfi Zadeh (1923) :** He introduced fuzzy sets and logic theory. 'As the complexity of a system increases, our ability to make precise and significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics ..... '. A corollary principle may be stated succinctly as "The closer one looks at a real-world problem, the fuzzier becomes its solution".

1.8 Matbematical modeling of fuzzy sets

The mathematical modeling of fuzzy concepts was presented by Zadeh in 1965. His contention is that meaning in natural language is a mater of degree. If we have a proposition such as "Ram is old", then it is not always possible to assert that it is either true or false. When we know that Ram's age is x, then the 'truth', or more correctly, the "compatibility" of x with "is old" is a matter of degree. It depends on our understanding of the concept "old". If the proposition is "Ram is under 50 years old" and we know Ram's age, then we can give a yes or no answer to whether the proposition is true or not. This can be formalized a fit by considering possible ages to be the interval $[0, \infty)$, letting A be the subset $\{x : x \in [0, \infty) \textit{ and } x < 22\}$, and then determining whether or not Ram's age is in A. But "old" cannot be defined as an ordinary subset of $[0, \infty)$. This led Zadeh was led to the notion of fuzzy subset. Clearly, 60 and 70 years olds are old, but with different degree as 70 is older than 60. This suggests that membership in a fuzzy subset should not be on a 0 or 1 basis, but rather on a 0 to 1 scale. So the membership should be an element of the interval [0, 1].

An ordinary subset A of a set X is determined by its characteristic function χ_A defined by

$$\chi_A(x) = \begin{cases} 1; \text{ if } x \in A \\ 0; \text{ if } x \notin A \end{cases}$$

The characteristic function of the subset A of the universal set X specifies whether or not an element is in A. If the value of the function is 1 then the element is in A and if the value is 0 then the element is not in A. There is only two possibility either the element is in A or is not in A. The characteristic function can take only two possible values 0 and 1 i.e. the range of the characteristic function is $\{0, 1\}$. This notion is generalized by allowing range of the function to be the closed interval [0, 1]. This generalized function of the characteristic function is called membership function and is denoted by $\mu_{\tilde{A}}(x)$ and the corresponding fuzzy subset will be denoted by $\tilde{A}$. Thus, $\mu_{\tilde{A}} : X \longrightarrow [0, 1]$ whereas $\chi_A : X \longrightarrow \{0, 1\}$.

Hence, the functions whose images are contained in $\{O, 1\}$ correspond to ordinary or crisp subset of X and the functions whose images are contained in [0, 1] correspond to fuzzy subset of X. It is common to refer a fuzzy subet simply as a fuzzy set, henceforth we also will do that.

Let us again consider the set "old persons". Here "old" is not well-defined in the sense of classical mathematics and cannot be precisely measured.

If we know that age of Ram is 55 years, it is not clear that Ram is old, also it is not clear whether Ram is old if his age is 49 years or 61 years. In classical set theory, we may draw a line at the exact age of say 80. As a result, a person who is exactly 80 years old belongs to the set and is considered to be "old" but another person of one-day less than 80 years will not be considered "old". This distinction is mathematically correct, but practically unreasonable. So we need to quantify the concept "old". Instead of a sharp cut at the exact age of 80, we use common sense and say "absolutely old" persons are those who are 80 year old or older and say "absolutely young" persons are those who are 30 years old or younger. All the other persons are old as well as young at the same time, with different degrees of oldness and youngness depending on their actual ages. Thus, the membership function of the fuzzy set $\tilde{A} = \{old\ persons\}$ may be defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0; & \text{if } x \leq 30 \\ \frac{x-30}{50}; & \text{if } 30 < x < 80 \\ 1; & \text{if } x \geq 80 \end{cases}$$

The graph of $\mu_{\tilde{A}}$ is given in Fig. 1.3.

A person of 55 years old is considered to be "old" with degree 0.5 and at the same time 'young' with degree 0.5. A person of 40 years old is considered to be "old" with degree $\frac{1}{5}$ and 'young' with degree $\frac{4}{5}$. We note that a person of age 30 yrs to 80 yrs is neither a member of $\tilde{A}$ fully, nor he is non-member of $\tilde{A}$ fully. He has a partial membership to the fuzzy set $\tilde{A}$.

Depending on the concept of "old" the membership function $\mu_{\tilde{A}}(x)$ will change. It may be linear as well as non-linear. So the set $\tilde{A}$ can have infinite possible membership function $\mu_{\tilde{A}}(x)$. Figs. 1.4 and 1.5 show two other $\mu_{\tilde{A}}(x)$. Let $\tilde{B}$ be the fuzzy set "young persons". We take the universal set as persons of all ages i.e. set of all positive real numbers. The membership function $\mu_{\tilde{B}}(x)$ may be defined as

$$\mu_{\tilde{B}}(x) = \begin{cases} 1; & \text{if } x \leq 30 \\ \frac{40-x}{10}; & \text{if } 30 < x < 40 \\ 1; & \text{if } x \geq 40 \end{cases}$$

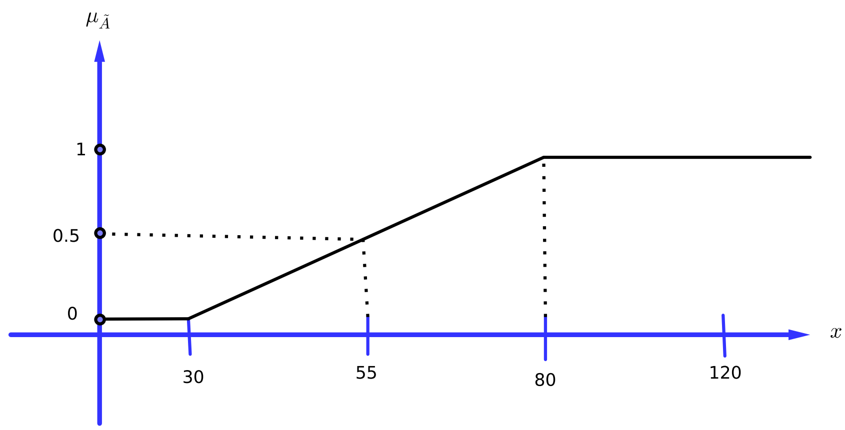

Figure 1.3: Membership function of old persons.

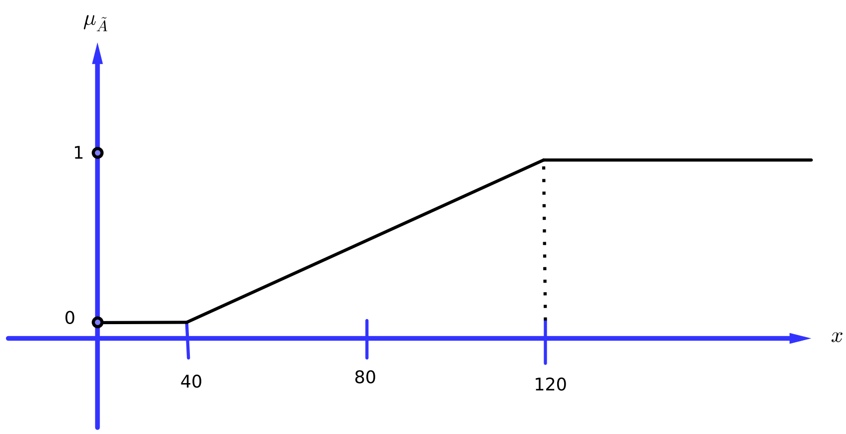

Figure 1.4: Membership function of old persons.

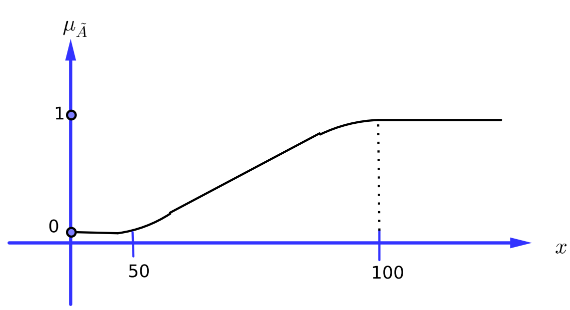

Figure 1.5: Membership function of old persons.

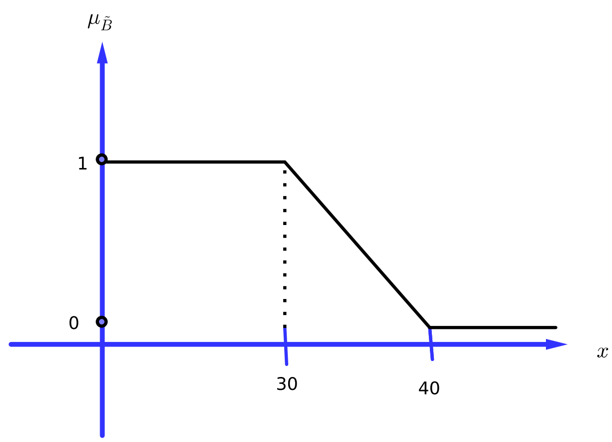

Figure 1.6: Membership function of old persons.

Fig. 1.6 shows the graph of this function. Another membership function of $\tilde{B}$ may be taken as

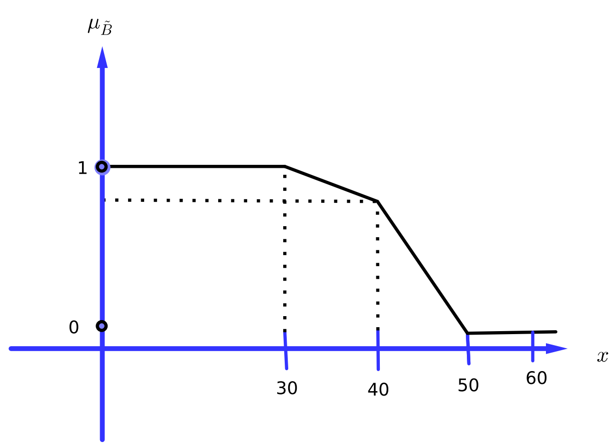

Figure 1.7: Membership function of old persons.

$$\mu_{\tilde{B}}(x) = \begin{cases} 1; & \text{if } x \le 30 \\ \frac{60-x}{30}; & \text{if } 30 < x < 40 \\ \frac{50-x}{15}; & \text{if } 40 \le x \le 50 \\ 0; & \text{if } x > 50 \end{cases}$$

Fig. 1.7 shows the graph of this function. It is piecewise linear.

Let us consider another fuzzy set $\tilde{C}$= "real numbers close to 5". One membership function of this fuzzy set $\tilde{C}$ is given by

$$\mu_{\tilde{C}}(x) = \begin{cases} 0; & \text{if } x \le 4.09 \\ \frac{x-4.09}{0.01}; & \text{if } 4.09 < x < 5 \\ \frac{5.01-x}{0.01}; & \text{if } 5 \le x \le 5.01 \\ 0; & \text{if } x > 5.01 \end{cases}$$

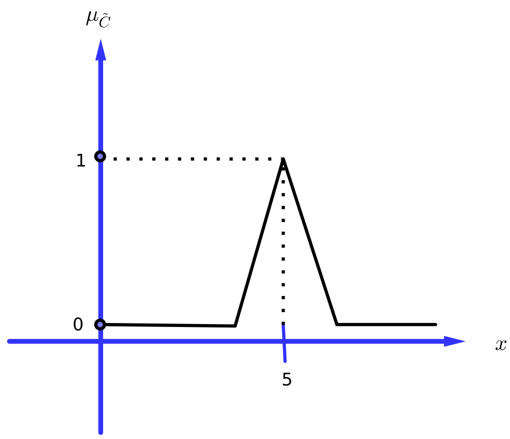

Figure 1.8: Membership function of old persons.

The graph of this function is shown in Fig. 1.8

Another membership function (non-linear) of this fuzzy set $\tilde{C}$ is given in Fig. 1.9. One more membership

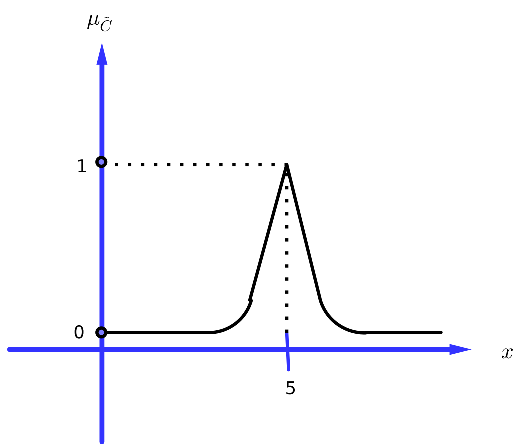

Figure 1.9: Membership function of old persons.

function of the fuzzy set may be taken as

$$\mu_{\tilde{C}}(x) = \frac{1}{1 + (x - 5)^2}$$

The graph of this function is given in Fig. 1.10.

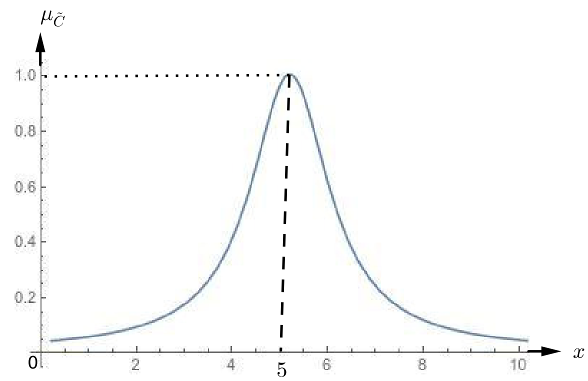

Figure 1.10: Membership function of old persons.

1.9 Fuzzy sets with a finite support

So far we have considered fuzzy sets on infinite support i.e. with the universal set as infinite set. Now, we consider situations where the universal set is a finite set. Let the finite universal set be $X = \{x_1, x_2, x_3, ..., x_n\}$. Let $\tilde{A} \subset X$ and grade of membership of $x_i \in \tilde{A}$ be a_i. Then the fuzzy set $\tilde{A}$ is expressed by the notation

$$\{(x_i,\, a_i) \in \tilde{A} \times [0,\, 1] \subset X \times [0,\, 1]\}$$

Here $a_i = \mu_{\tilde{A}}(x_i)$, so the notation becomes

$$\{(x_i,\, \mu_{\tilde{A}}(x_i)) \in \tilde{A} \times [0,\, 1] \subset X \times [0,\, 1]\}$$

Often in the literature the following notation is used

$$\tilde{A} = a_1/x_1 + a_2/x_2 + a_3/x_3 + ... + a_n/x_n$$

i.e, $\tilde{A} = \mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 + \mu_{\tilde{A}}(x_3)/x_3 + ... + \mu_{\tilde{A}}(x_n)/x_n$.

Here, the slash is employed to link the elements of the support with their grades of membership in $\tilde{A}$, and the plus sign indicates, rather than any sort of algebraic addition, that the listed pairs of elements and membership grades collectively form the definition of the set $\tilde{A}$. For the case in which a fuzzy set $\tilde{A}$ is defined on a universal set that is finite or countable, we may write, respectively,

$$\tilde{A} = \sum_{i=1}^{n} a_i/x_i, \text{ or } \tilde{A} = \sum_{i=1}^{\infty} a_i/x_i$$

i.e. $\tilde{A} = \sum_{i=1}^{n} \mu_{\tilde{A}}(x_i)/x_i$, or $\tilde{A} = \sum_{i=1}^{\infty} \mu_{\tilde{A}}(x_i)/x_i$.

1.10 Example

Let us consider the fuzzy set $\tilde{A}$ consisting of six ordered pairs as

$$\tilde{A} = \{(x_1, 0.2), (x_2, 1), (x_3, 0.8), (x_4, 0.3), (x_5, 0.5), (x_6, 0.1)\}.$$

The elements x_i, $i = 1, 2, ...6$ are not necessary numbers. They belong to the classical set $\{x_1, x_2, ..., x_6\}$ which is a subset of a certain universal set X. Here, the membership function $\mu_{\tilde{A}}(x_i)$ of $\tilde{A}$ takes the following values on $[0, 1]$. $\mu_{\tilde{A}}(x_1) = 0.2$, $\mu_{\tilde{A}}(x_2) = 1$, $\mu_{\tilde{A}}(x_3) = 0.8$, $\mu_{\tilde{A}}(x_4) = 0.3$, $\mu_{\tilde{A}}(x_5) = 0.5$, $\mu_{\tilde{A}}(x_1) = 0.1$.

The following interpretation could be given to $\mu_{\tilde{A}}(x_i)$, $i = 1, 2, ...6$. The element x_2 is a full member of the fuzzy set $\tilde{A}$, while the element x_6 is a member of $\tilde{A}$ a little, x_1 and x_4 are a little more members of$\tilde{A}$, the element x_3 is almost a full member of $\tilde{A}$, while x_5 is more or less a member of $\tilde{A}$.

Now we specify in two different way the element x_i in $\tilde{A}$.

(i) First, we assume that x_i are integers e.g. $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$, $x_5 = 5$, $x_6 = 6$; they belong to the set $A = \{1, 2, 3, 4, 5, 6\}$, a subset of the universe $N = \{1, 2, 3, ..., \infty\}$. The fuzzy set $\tilde{A}$ then becomes

$$\tilde{A} = \{(1, 0.2), (2, 1), (3, 0.8), (4, 0.3), (5, 0.5), (6, 0.1)\}$$

The membership function $\mu_{\tilde{A}}(x)$ is shown in Fig.

(ii) Secondly, let us consider the universal set X as "All friends of Ram" & $\tilde{A}$ be the set "close friends of Ram".

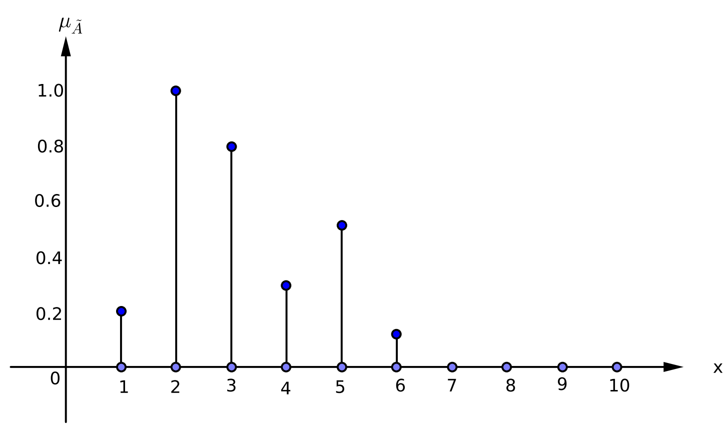

Figure 1.11: Membership function of old persons.

Let $\tilde{A} = \{(Rahim, 0.2), (Jadu, 1), (Kamal, 0.8), (Bimal, 0.3), (Amal, 0.5), (Tapan, O.l)\}$ Here $x_1 = Rahim$, $x_2 = Jadu$, $x_3 = Kamal$, $x_4 = Bimal$, $x_5 = Amal$ and $x_6 = Tapan$.

We note that regarding closeness in friendship of Ram : Jadu is closest, Tapan is little close, Rahim and Bimal are a little more close, Kamal is almost close, while Amal is more or less close.

1.11 Summary

In this module we have introduced the notion of fuzzy sets, its necessity, its application area. Also we have discussed the concept of uncertainty and its types. Finally, the mathematical modeling of fuzzy sets is done.

1.12 Suggested Further Readings

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