**Lecture 1: Concept of Classical Mechanics**
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**History of the formation of classical mechanics:**

The ancient civilizations of Mesopotamia, Egypt and the Indus Valley all demonstrated an understanding of the motion of the sun, moon and stars; they could even predict the dates of eclipses by the 18th century B.C. The stars and planets were often a target of worship of this civilization and they believed that the stars and planets represent their gods. Such supernatural explanations by definition lacked evidence, but the records of observation laid the foundation for generations of observers. **Celestial mechanics** thus became the study of how things move about the heavens. The ancient Greeks were the first to consistently seek natural (as opposed to supernatural) explanations. Philosophers like Thales (624-545 B.C.) rejected non-naturalistic explanations for natural phenomena and proclaimed that every event had a natural cause. A Greek philosopher, Aristotle ($4^{th}$ century BC), founder of Aristotelian physics, may have been the first to maintain the idea that "everything happens for a reason" and that theoretical principles can assist in understanding the nature. The Aristotelian mechanics is the main theory of mechanics in antiquity. A later developer in this tradition is Hipparchus. Aristotle argued, in 'On the Heavens', that terrestrial bodies rise or fall to their "natural place" and stated as a law the correct approximation that an object's speed of fall is proportional to its weight and inversely proportional to the density of the fluid it is falling through. Aristotle believed in logic and observation but it would be more than eighteen hundred years before Francis Bacon would first develop the scientific method of experimentation, which he called a 'vexation of nature'. Aristotle was the first to approach something similar to the law of inertia. In 260 BC, Archimedes works out the principle of the lever and connects buoyancy to weight. In 60, Hero of Alexandria writes *Metrica*, *Mechanics* (on means to lift heavy objects), and *Pneumatics* (on machines working on pressure). However, the myriad explanations involving, for example, "bodily humors" and "cosmic shells surrounding Earth," were indeed naturalistic, but most were fantastically wrong. A particularly tenacious set of wrong ideas centered on motion, which for nearly 2000 years built on the work of Aristotle (384-322 B.C.). This work, dubbed "the theory of impetus," would undergo major revisions in the sixth, 12th and 14th centuries A.D. **Terrestrial mechanics** thus became the study of how things move and interact on the Earth's surface. Many of these preserved ideas come forth as eminently reasonable, but there is a conspicuous lack of both mathematical theory and controlled experiment. These latter became decisive factors in forming modern science, and their early application came to be known as **classical mechanics**.

In medieval age, Persian Islamic polymath Ibn Sīnā published his theory of motion in *The Book of Healing* in 1020. He said that an impetus is imparted to a projectile by the thrower, and viewed it as persistent, requiring external forces such as air resistance to dissipate it. He made distinction between 'force' and 'inclination' (called "mayl"), and argued that an object gained mayl when the object is in opposition to its natural motion. This conception of motion is consistent with the Newton's first law of motion, inertia which states that an object in motion will stay in motion unless it is acted on by an external force. In 1021, Al-Biruni uses three orthogonal coordinates to describe point in space. In the 12th century, Hibat Allah Abu'l-Barakat al-Baghdaadi adopted and modified Avicenna's theory on projectile motion. In his *Kitab al-Mu'tabar,* Abu'l-Barakat stated that the mover imparts a violent inclination (*mayl qasri*) on the moved and that this diminishes as the moving object distances itself from the mover. According to Shlomo Pines, al-Baghdaadi's theory of motion was "the oldest negation of Aristotle's fundamental dynamic law (a constant force produces a uniform motion) and is thus an anticipation in a vague fashion of the fundamental law of classical mechanics (a force applied continuously produces acceleration). The same century, Ibn Bajjah proposed that for every force there is always a reaction force. But he did not specify that these forces be equal, it is the Newton's third law of motion which states that for every action there is an equal and opposite reaction. In the 14th century, French priest Jean Buridan developed the theory of impetus, influenced by Ibn Sina and al-Baghdaadhi. Albert, Bishop of Halberstadt, developed the theory further. In his *Elementa super demonstrationem ponderum*, a 13th-century European mathematician and scientist Jordanus de Nemore introduced the concept of "positional gravity" and the use of component forces.

During the early modern period, some philosophers, astronomers, scientist and mathematician have their contribution in developing the different fields in science, among them Galileo, Newton, Kepler and Copernicus laid the foundation for classical mechanics. The first published causal explanation of the motions of planets was Johannes Kepler's *Astronomia nova* published in 1609. He concluded, based on Tycho Brahe's observations of the orbit of Mars, that the orbits were ellipses. Tycho Brahe (1546-1601) was one of the first astronomers to use clocks capable of counting minutes and seconds, along with quadrants and sextants, to track the movements of celestial objects. The first of the Kepler's laws of planetary motion, published in his 1609 work, "Astronomia Nova," showed that planets move in elliptical paths around the sun. Adopting Copernicus's heliocentric hypothesis, Galileo believed the Earth was the same as other planets. Galileo was proposing the abstract mathematical laws for the motion of objects. He may (or may not) have performed the famous experiment of dropping two cannonballs of different weights from the tower of Pisa, the theory and the practice of this experiment showed that they both hit the ground at the same time. The reality of this experiment is disputed, but, more importantly, he did carry out quantitative experiments by rolling balls on an inclined plane. His correct theory of accelerated motion was apparently derived from the results of such experiments, and forms a cornerstone of classical mechanics. Galileo also found that a body dropped vertically hits the ground at the same time as a body projected horizontally. According to Galileo, the resistance of the air exhibits itself in two ways: first by offering greater impedance to less dense than to very dense bodies, and secondly by offering greater resistance to a body in rapid motion than to the same body in slow motion. Galileo Galilei published the first mathematical proof that uniform acceleration would cause projectiles to move in parabolic trajectories that matched observations, thus showing that terrestrial mechanics are governed by mathematics. Similarly, and also in the 16th century, celestial mechanics was shown to have extremely strong ties to mathematics. Newton founded his principles of natural philosophy on three proposed laws of motion: the law of inertia, the law of acceleration, and the law of action and reaction; and hence laid the foundation for classical mechanics. He was the first to prove these laws of motion govern both earthly and celestial objects. The proper scientific and mathematical treatment on the Newton's second and third laws was given in Newton's *Philosophiæ Naturalis Principia Mathematica*. In mechanics, Newton was also the first to provide the first correct scientific and mathematical formulation of gravity in the Newton's law of universal gravitation. Newton built on the work of Galileo and Kepler to show that the elliptical movements of the celestial realm and parabolic movements of the terrestrial realm could be explained by one elegant mathematical law, his Law of Universal Gravitation. In addition, he formalized the laws of motion by describing them in the language of mathematics. The combination of Newton's laws of motion and gravitation provide the fullest and most accurate description of classical mechanics. He demonstrated that these laws apply to everyday objects as well as to celestial objects. In particular, he obtained a theoretical explanation of Kepler's laws of motion of the planets. Newton had previously developed the calculus of mathematics, which is necessary to perform the mathematical calculations involved in classical mechanics. For acceptability, his book *Principia* was formulated entirely in terms of the long-established geometric methods, which were soon eclipsed by his calculus. However it was Gottfried Leibniz who, independently of Newton, developed a calculus with the notation of the derivative and integral which are used to this day. Classical mechanics retains Newton's dot notation for time derivatives. Newton also enunciated the principles of conservation of momentum and angular momentum. Leonhard Euler extended Newton's laws of motion from particles to rigid bodies with two additional laws. The idea was articulated by Euler (1727), and in 1782 Giordano Riccati began to determine elasticity of some materials, followed by Thomas Young. Newton and most of his contemporaries, with the notable exception of Huygens, worked on the assumption that classical mechanics would be able to explain all entities, including light in the form of geometric optics. Even when discovering the so-called Newton's rings, a wave interference phenomenon, he maintained his own corpuscular theory of light but avoided wave principles for explaining the Newton’s rings. The classical mechanics is often referred to as Newtonian mechanics, because nearly the entire study builds on the work of Isaac Newton. Classical mechanics grew throughout the 18th and 19th centuries to describe everything from optics, fluids and heat to pressure, electricity and magnetism. After Newton, classical mechanics became a principal field of study in mathematics as well as physics. Several re-formulations progressively allowed finding solutions to a far greater number of problems. The first notable re-formulation was constructed in 1788 by Joseph Louis Lagrange, an Italian-French mathematician. In Lagrangian mechanics the solution uses the path of least action and follows the calculus of variations. Lagrangian mechanics was in turn re-formulated in 1833 by William Rowan Hamilton. The advantage of Hamiltonian mechanics was that its framework allowed a more in-depth look at the underlying principles. Lagrangian and Hamiltonian extend substantially beyond Newton's work, particularly through their use of analytical mechanics.

At present time, i.e. the time between the end of the 20th century and starting of 21st century, the place of classical mechanics in physics has been no longer that of an independent theory. Along with classical electromagnetism, the classical mechanics has now considered an approximate theory to the more general quantum mechanics and become imbedded in relativistic quantum mechanics or quantum field theory. Emphasis has shifted to understanding the fundamental forces of nature as in the Standard model and its more modern extensions into a unified theory of everything. Classical mechanics has also been a source of inspiration for mathematicians. The realization that the phase space in classical mechanics admits a natural description as a symplectic manifold (indeed a cotangent bundle in most cases of physical interest), and symplectic topology, which can be thought of as the study of global issues of Hamiltonian mechanics, has been a fertile area of mathematics research since the 1980s. Classical mechanics is a theory useful for the study of the motion of non-relativistic, non-quantum mechanical, low-energy and massive particles in weak gravitational fields. It can also be defined as a branch of science which deals with the motion of and forces on bodies not in the quantum realm. In the 21st century classical mechanics has been extended into the complex domain and complex classical mechanics exhibits behaviors very similar to quantum mechanics. However, the field of science is today less widely understood in terms of quantum theory.

**Classical mechanics:**

Mechanics is the study of motion of physical bodies. The possible and actual motion of physical objects, whether large or small, falls under the domain of mechanics. The motions of celestial bodies (planets, stars, etc.) path of an artillery shell, or of a space-satellite sent from earth to a planet are among its problems. There are two sub disciplines, namely classical mechanics and quantum mechanics.

The physics dealing with the mathematical study of the motion of everyday objects on a large scale is known as classical mechanics. The motion can be a linear type, oscillating type, rotational, rolling, circular, and comparative different sorts of motion experienced in nature. In classical mechanics, the objects in question are neither too big so that a close agreement between theory and experiment is desirable, nor too small interacting objects or systems of an atomic scale. It is estimation for the laws of nature and gives great outcomes on the macroscopic scale.

The following introduces the **basic concepts of classical mechanics**. For simplicity, it often models real-world objects as point particles (objects with negligible size). The motion of a point particle is characterized by a small number of parameters: its position, mass, and the forces applied to it. Each of these parameters is discussed later. In reality, **the kind of objects that classical mechanics can describe always has a non-zero size**. Objects with non-zero size have more complicated behavior than hypothetical point particles, because of the additional degrees of freedom: a baseball can spin while it is moving, for example. However, the results for point particles can be used to study such objects by treating them as composite objects, made of a large number of collectively acting point particles. The center of mass of a composite object behaves like a point particle.

The study of the motion of bodies is an ancient one, making classical mechanics one of the oldest and largest subjects in science, engineering and technology. Classical mechanics is frequently called Newtonian mechanics because of the fact that almost the entire study is crafted by Isaac Newton**,** though textbook authors often consider Newtonian mechanics as one of the three main formalisms of classical mechanics, along with Lagrangian mechanics and Hamiltonian mechanics. The earliest development of classical mechanics is associated with the physical concepts employed by and the mathematical methods invented by Newton, Leibniz, and others. Later, more abstract and general methods were developed, leading to reformulations of classical mechanics known as Lagrangian mechanics and Hamiltonian mechanics. These advances were largely made in the 18th and 19th centuries, and they extend substantially beyond Newton's work, particularly through their use of analytical mechanics.

Classical mechanics describes the motion of macroscopic objects, from projectiles to parts of machinery, as well as astronomical objects, such as spacecraft, planets, stars, and galaxies. Classical mechanics has thoughts of particles, their positions, energy, momentum, and motion in space and time. It claims to predict definite position, speed, momentum, energy, and other such properties of a framework at some given point of time if its underlying state and the forces acting on it are known. For examples, Using just a few equations, scientists can describe the motion of a body flying through the air, the pull of a magnet, and forecast eclipses of the moon.

Classical mechanics uses common-sense notions of how matter and forces exist and interact. It assumes that matter and energy have definite, knowable attributes such as where an object is in space and its speed. Non-relativistic mechanics also assumes that forces act instantaneously. Some mathematical laws and principles at the core of classical mechanics include the following:

(i) **Newton's First Law of Motion**: A body at rest will remain at rest, and a body in motion will remain in motion unless it is acted upon by an external force.

(ii) **Newton's Second Law of Motion**: The net force acting on an object is equal to the mass of that object times its acceleration.

(iii) **Newton's Third Law of Motion**: For every action, there is an equal and opposite reaction.

(iv) **Newton's Law of Universal Gravitation**: The pull of gravity between two objects will be proportional to the masses of the objects and inversely proportional to the square of the distance between their centers of mass.

(v) **Law of Conservation of Energy**: Energy cannot be created nor destroyed, and instead changes from one form to another; for example, mechanical energy turning into heat energy.

(vi) **Law of Conservation of Momentum**: In the absence of external forces such as friction, when objects collide, the total momentum before the collision is the same as the total momentum after the collision.

(vii) **Bernoulli's Principle**: Within a continuous streamline of fluid flow, a fluid's hydrostatic pressure will balance in contrast to its speed and elevation.

**Frame of reference:**

While the position, velocity and acceleration of a particle can be described with respect to any observer in any state of motion, classical mechanics assumes the existence of a special family of reference frames in which the mechanical laws of nature take a comparatively simple form. These special reference frames are called **inertial frames**. An inertial frame is one that when an object that has no force interactions (an idealized situation) is viewed from that frame, it appears either to be at rest or in a state of uniform motion in a straight line. **This is the fundamental definition of an inertial frame**. They are characterized by the requirement that all forces entering the observer's physical laws originate from identifiable sources caused by fields, such as **electro-static field** (caused by static electrical charges), **electro-magnetic field** (caused by moving charges), **gravitational field** (caused by mass), and so forth. A **non-inertial reference frame** is one that is accelerating with respect to an inertial frame. The non-inertial frame form the basis for Einstein's relativity, and in such a non-inertial frame a particle appears to be acted on by other forces not explained by existing fields. Such other forces are called variously fictitious forces, inertia forces, or pseudo-forces. Hence, it appears that there are other forces that enter the equations of motion solely as a result of the relative acceleration.

Consider two reference frames $S$ and $S'$. For observers in each of the reference frames an event has space-time coordinates of ($x$, $y$, $z$, $t$) in frame $S$ and ($x'$, $y'$, $z'$, $t'$) in frame $S'$. Assuming time is measured the same in all reference frames, and if we require $x = x'$ when $t = 0$, then the relation between the space-time coordinates of the same event observed from the reference frames $S'$ and $S$, which are moving at a relative velocity of $u$ in the $x$ direction is:

$$x' = x - ut$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

This set of formulas defines a group transformation known as the Galilean transformation (informally, the ***Galilean transform***).

The transformations have the following consequences:

(i) $\boldsymbol{v}' = \boldsymbol{v} - \boldsymbol{u}$ (the velocity $\boldsymbol{v}'$ of a particle from the perspective of $S'$ is slower by $\boldsymbol{u}$ than its velocity $\boldsymbol{v}$ from the perspective of $S$)

(ii) $\boldsymbol{a}' = \boldsymbol{a}$ (the acceleration of a particle is the same in any inertial reference frame)

(iii) $\boldsymbol{F}' = \boldsymbol{F}$ (the force on a particle is the same in any inertial reference frame)

(iv) The speed of light is not a constant in classical mechanics

**Mechanics of a particle or a system of particles:**

We shall study some physical quantity and laws for a particle or a system of particles in motion or in rest using Newtonian mechanics.

**Position:**

The *position* of a point particle is defined with respect to an arbitrary fixed reference point in space called the origin O, in space, to which is attached a coordinate system, say X-Y. A simple coordinate system, the position of a point P with respective to O is described by means of an arrow designated as **r**, which is called a position vector in classical mechanics. The position **r** is pointing from the origin O to the point particle P.

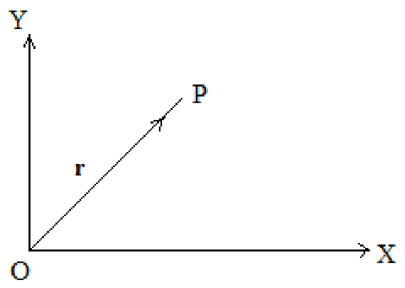


**Fig. 1: Position of a particle P in X-Y coordinate system**

From Fig. 1, we can write

$$OP = \boldsymbol{r}$$

In general, the point particle need not be stationary relative to O, such that **r** is a function of time (t). That means **r** can be expressed as

$$\boldsymbol{r} = \boldsymbol{r}(t) \qquad \dots (1)$$

Suppose that we have a team of observers who continually report the location of a body to us as time progresses, assuming the body is on a horizontal line (Fig. 2). To be more exact, our observers report the position $x$ of the body from some arbitrarily chosen reference point O, the origin. A $+x$ value implies that the body is located $x$ unit to the right of the origin O, whereas a negative $-x$ value implies that the body is located $|x|$ unit to the left of the origin.

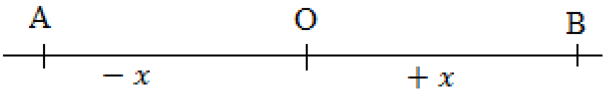


**Fig. 2: A body is on a horizontal line**

**Displacement:**

Displacement of a particle moving in a straight line is the difference between final and initial positions of the particle. It is a vector quantity. If the particle moves from the position $x(t_1)$ to the position $x(t_2)$, then its displacement is $x = x(t_2) - x(t_1)$ over the time interval $[t_1, t_2]$, or over the time of $t = t_2 - t_1$. This is shown in Fig. 3. In particular, the position of a particle is its displacement from the origin. Like the position, the displacement is a vector quantity. A $+x$ value of the displacement implies that the body is located $x$ unit to the right of the origin O, whereas a negative $-x$ value of the displacement implies that the body is located $|x|$ unit to the left of the origin.

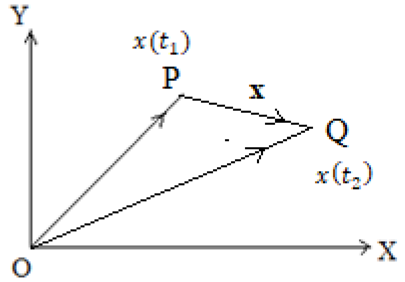


**Fig. 3: Position of a particle at $t_1$ and $t_2$**

**Speed, velocity and acceleration:**

Speed, velocity, and acceleration are all related to each other, though they represent different measurements. The definition of speed, velocity and acceleration are as follows:

**Speed: Speed** is a scalar quantity that indicates the rate of motion distance per time. Its unit is length per time. Put another way, speed is a measure of distance traveled over a certain amount of time. Speed is often described simply as the distance traveled per unit of time. Speed measures distance, a scalar quantity that measures the total length of an object's path. It is used to understand how fast an object is moving.

If a body travel a total distance of $d$ over a time of $t$, then the speed $s$ of the body can be expressed mathematically as

$$s = \frac{d}{t} \qquad \dots (2)$$

**Velocity: Velocity** is the rate of change of displacement with time. It is a vector measurement of the rate and direction of motion. Velocity is a vector quantity that indicates displacement, time, and direction. Velocity measures displacement, a vector quantity indicating the difference between an object's final and initial positions. So, velocity is a measure of displacement per unit time along a straight line.

If the displacement of the body is $x$ at a time $t$, then the velocity $v$ of the body can be expressed mathematically as

$$v = \frac{x}{t}$$

The Fig. 3 showing the displacement of the particle is $x = x(t_2) - x(t_1)$ over the time of $t = t_2 - t_1$. The velocity of that particle can be expressed as

$$v = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{x}{t} \qquad \dots (3)$$

Velocity measures motion of a body starting in one place and heading toward another place. The practical applications of velocity are endless, but one of the most common reasons to measure velocity is to determine how quickly the body or anything in motion will arrive at a destination from a given position. Velocity makes it possible to create timetables for travel, a common type of physics problem assigned to students. For example, if a train leaves a railway station at 1 $pm$ and you know the velocity at which the train is moving north, you can predict when it will arrive at another station.

How should we choose the time interval $\Delta t = t_2 - t_1$ appearing in expression (3) ? Obviously, in the simple case in which the body is moving with constant velocity, we can make $\Delta t$ as large or small as we like, and it will not affect the value of $v$. Suppose, however, that +$v$ $v$ is constantly changing in time, as is generally the case. In this situation, $\Delta t$ must be kept sufficiently small that the body's velocity does not change appreciably between times $t_1$ and $t_2 = t_1 + \Delta t$. If $\Delta t$ is made too large then formula (3) becomes invalid.

Suppose that we require a general expression for instantaneous velocity which is valid irrespective of how rapidly or slowly the body's velocity changes in time. We can achieve this goal by taking the limit of equation (3) as $\Delta t$ approaches zero, i.e., $\Delta t \to 0$. This ensures that no matter how rapidly $v$ varies with time, the velocity of the body is always approximately constant in the interval $t_1$ and $t_2 = t_1 + \Delta t$. Thus, if $\Delta x = x(t_2) - x(t_1)$, then

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad \text{... (4)}$$

where $\frac{dx}{dt}$ represents the derivative of $x$ with respect to $t$. The above definition is particularly useful if we can represent $x(t)$ as an analytic function of time $t$, because it allows us to immediately evaluate the instantaneous velocity $v(t)$ via the rules of calculus. Thus, if $x(t)$ is given by formula (4), then

$$v = \frac{dx}{dt} \qquad \text{... (5)}$$

In classical mechanics, velocities are directly additive and subtractive. For example, if one car traveling east at 60 km/h passes another car traveling east at 50 km/h, then from the perspective of the slower car, the faster car is traveling east at 60 − 50 = 10 km/h. Whereas, from the perspective of the faster car, the slower car is moving 10 km/h to the west. Velocities are directly additive as vector quantities; they must be dealt with using vector analysis.

Mathematically, if the velocity of the first object is denoted by the vector $\boldsymbol{u}$ and the velocity of the second object by the vector $\boldsymbol{v}$, assuming these two objects moving in the same direction, then the velocity of the first object as seen by the second object is

$$u' = u - v$$

Similarly,

$$v' = v - u$$

It may be noted here that the terms velocity and speed are often confused with one another. A velocity can be either positive or negative, depending on the direction of motion. The conventional definition of ***speed*** is that it is the magnitude of velocity (***i.e.***, it is $v$ with the sign stripped off). It follows that a body can never possess a negative speed.

**Angular velocity:**

Angular velocity of a particle is a velocity which is measured in angle per unit time. It is observed when a particle revolves in a circular path about a fixed origin. If the tangential velocity of the revolving particle is $v$ and the radius of the circle is $r$, then the angular velocity$\omega$ of the particle can be written as

$$\omega = \frac{v}{r}$$

In physics, **angular velocity** refers to how fast an object rotates or revolves relative to another point, i.e. how fast the angular position or orientation of an object changes with time. There are two types of angular velocity: orbital angular velocity and spin angular velocity. Spin angular velocity refers to how fast a rigid body rotates with respect to its centre of rotation. Orbital angular velocity refers to how fast a point object revolves about a fixed origin, i.e. the time rate of change of its angular position relative to the origin. Spin angular velocity is independent of the choice of origin, in contrast to orbital angular velocity which depends on the choice of origin. By convention, positive angular velocity indicates counter-clockwise rotation, while negative is clockwise.

**Angular momentum:** Angular momentum is the analogue of linear momentum in the case of rotational motion. **Angular momentum** $L$ of a particle of mass $m$ (relative to a particular origin) is defined by

$$L = r \times p = r \times mv = mvr = m\omega^2 r$$

**Torque** is a measure of the force that can cause an object to rotate about an axis. In other word, the torque is the moment of the force about the origin. Just as force is what causes an object to accelerate in linear kinematics, torque is what causes an object to acquire angular acceleration. Torque $\tau$ exerted by a force $F$ (relative to a particular origin) is defined by

$$\tau = r \times F$$

where, $r$ is the distance perpendicular to the applied force $F$. Torque is a vector quantity. The direction of the torque vector depends on the direction of the force on the axis.

It may be noted that the angular velocity, angular momentum, and torque depend on the choice of origin.

**Acceleration: Acceleration** is defined as the rate of change of velocity with time. It is a vector quantity and it has dimensions of length and time over time. Acceleration is often referred to as "speeding up", but it really measures changes in velocity. Acceleration can be experienced every day in a vehicle. You step on the accelerator and the car speeds up, increasing its velocity.

Another term is “deceleration”; it is related to the acceleration of a body and is opposite to the velocity or acceleration. Deceleration is the rate at which a body slows down. Deceleration is the final velocity minus the initial velocity, with a negative sign in the result because the velocity is dropping. It has dimensions of length and time over time

If the velocity of a body is $v$ at a time $t$, then the acceleration $a$ of the body can be expressed mathematically as

$$a = \frac{v}{t}$$

If $\Delta v$ is the change in velocity of a body between time interval of $t_1$ and $t_2 = t_1 + \Delta t$, then the definition of the acceleration $a$ implies that

$$a = \frac{\Delta v}{\Delta t} \qquad \ldots (6)$$

How should we choose the time interval $\Delta t$ appearing in equation (6) ? Again, in the simple case in which the body is moving with constant acceleration, we can make $\Delta t$ as large or small as we like, and it will not affect the value of $a$. Suppose, however, that $a$ is constantly changing in time, as is generally the case. In this situation, $\Delta t$ must be kept sufficiently small that the body's acceleration does not change appreciably between times $t_1$ and $t_2 = t_1 + \Delta t$.

A general expression for instantaneous acceleration, which is valid irrespective of how rapidly or slowly the body's acceleration changes in time, can be obtained by taking the limit of equation (6) as $\Delta t$ approaches zero:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad \ldots (7)$$

The above definition is particularly useful if we can represent $x(t)$ as an analytic function of time $t$, because it allows us to immediately evaluate the instantaneous acceleration $a(t)$ via the rules of calculus. Thus, if $v = \frac{dx}{dt}$, then the formula (7) can be written as

$$a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \qquad \ldots (4)$$

**Momentum:**

Momentum of a particle is the product of mass and velocity of it. It is a vector quantity and has a direction along the direction of the particle's velocity. The momentum of a particle of mass $m$ moving with velocity $v$ can be expressed as

$$p = mv$$

For example, a body of mass 1 $kg$ traveling at 1 $m/s$ in a straight line, it has a momentum of 1 $kg.m/s$.

According to Newton's second law of motion, the time-rate of change of momentum of a body is equal to the net force acting on it. This is the relation between force ($F$) and momentum of the body ($p$) and is given by

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} = ma$$

where, $a = \frac{dv}{dt}$ is the acceleration of the body.

The momentum of a system of particles is the vector sum of their momenta. If two particles have respective masses $m_1$ and $m_2$, and velocities $v_1$ and $v_2$, the total momentum is

$$\begin{aligned} p &= p_1 + p_2 \\ &= m_1 v_1 + m_2 v_2 \end{aligned}$$

The momenta of more than two particles can be added more generally with the following:

$$p = \sum_i m_i v_i$$

A system of particles has a center of mass ($r_{cm}$), a point determined by the weighted sum of their positions:

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum_i m_i r_i}{\sum_i m_i}$$

If one or more of the particles is moving, the center of mass of the system will generally be moving as well (unless the system is in pure rotation around it). If the total mass of the particles is $m$, and the center of mass is moving at velocity $v_{cm}$, the momentum of the system is:

$$p = mv_{cm}$$

This is known as Euler's first law.

Momentum depends on the frame of reference, but in any inertial frame it is a conserved quantity, meaning that if a closed system is not affected by external forces, its total linear momentum does not change. Momentum is also conserved in special relativity, electrodynamics, quantum mechanics, quantum field theory, and general relativity. It is an expression of one of the fundamental symmetries of space and time: translational symmetry. In Lagrangian and Hamiltonian mechanics, the conserved quantity is generalized momentum.

**Forces – Newton's second law:**

Newton was the first to mathematically express the relationship between force and momentum. Some physicists interpret Newton's second law of motion as a definition of force and mass, while others consider it a fundamental postulate, a law of nature. Either interpretation has the same mathematical consequences, historically known as "Newton's Second Law":

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt}$$

The quantity $m\mathbf{v}$ is called the (canonical) momentum. The net force on a particle is thus equal to the rate of change of the momentum of the particle with time. Since the definition of acceleration is $\mathbf{a} = d\mathbf{v}/dt$, the second law can be written in the simplified and more familiar form:

$$F = ma$$

So long as the force acting on a particle is known, Newton's second law is sufficient to describe the motion of a particle. Once independent relations for each force acting on a particle are available, they can be substituted into Newton's second law to obtain an ordinary differential equation, which is called the *equation of motion*.

Important forces include the gravitational force and the Lorentz force for electromagnetism. In addition, Newton's third law can sometimes be used to deduce the forces acting on a particle: if it is known that particle A exerts a force **F** on another particle B, it follows that B must exert an equal and opposite ***reaction force***, −**F**, on A. The strong form of Newton's third law requires that **F** and −**F** act along the line connecting A and B, while the weak form does not. Illustrations of the weak form of Newton's third law are often found for magnetic forces.

**Work and energy:**

If a constant force **F** is applied to a particle that makes a displacement Δ**r**, the *work done* by the force is defined as the scalar product of the force and displacement vectors:

$$W = F.\Delta r$$

More generally, if the force varies as a function of position as the particle moves from $\mathbf{r}_1$ to $\mathbf{r}_2$ along a path *C*, the work done on the particle is given by the line integral

$$W = \int_C F(r).dr$$

If the work done in moving the particle from $\mathbf{r}_1$ to $\mathbf{r}_2$ is the same no matter what path is taken, the force is said to be conservative. Gravity is a conservative force, as is the force due to an idealized spring, as given by Hooke's law. The force due to friction is non-conservative.

The kinetic energy $E_k$ of a particle of mass *m* travelling at speed *v* is given by

$$E_k = \frac{1}{2}mv^2$$

For extended objects composed of many particles, the kinetic energy of the composite body is the sum of the kinetic energies of the particles.