**ECO 202: Theories of Economic Growth**

**Group- B**

**Topic 2: Endogenous growth with technological change**

Endogenous growth with Technological Change: The Romer Model

The Solow model identified technological progress or improvements in total factor productivity (TFP) as the key determinant of growth in the long run, but did not provide any explanation of what determines it. In the technical language used by macroeconomists, long-run growth in the Solow framework is determined by something that is *exogenous* to the model.

Consider a particular model that makes technological progress *endogenous*, meaning determined by the actions of the economic agents described in the model. The model, due to Paul Romer (“Endogenous Technological Change,” *Journal of Political Economy*, 1990) starts by accepting the Solow model’s result that technological progress is what determines long-run growth in output per worker. But, unlike the Solow model, Romer attempts to explain what determines technological progress.

## TFP Growth as Invention of New Inputs

Romer describes the aggregate production function as

*Y* = *L*1*−α* (*xα* + *xα* + + *xα* ) = *L*1*−α* Σ *xα*

*A*

*Y*

1

2

*A*

*Y*

*i*

*i*=1

(1)

where *LY* is the number of workers producing output and the *xi*’s are different types of capital goods. The crucial feature of this production function is that diminishing marginal returns applies, not to capital as a whole, but separately to each of the individual capital goods (because 0 *< α <* 1).

If *A* was fixed, the pattern of diminishing returns to each of the separate capital goods would mean that growth would eventually taper off to zero. However, in the Romer model, *A* is not fixed. Instead, there are *LA* workers engaged in R&D and this leads to the invention of new capital goods.

This is described using a “production function” for the change in the number of capital goods:

*A*˙ = *γLλ Aφ* (2)

*A*

The change in the number of capital goods depends positively on the number of researchers (*λ* is an index of how slowly diminishing marginal productivity sets in for researchers) and also on the prevailing value of *A* itself. For instance, the invention of a new piece of software will have relied on the previous invention of the relevant computer hardware, which itself relied on the previous invention of semiconductor chips, and so on.

Romer’s model contains a full description of the factors that determines the fraction of workers that work in the research section. The research sector gets rewarded with patents that allow it to maintain a monopoly in the product it invents; wages are equated across sectors, so the research sector hire workers up to point where their value to it is as high as it is to producers of final output. In keeping with the spirit of the Solow model, the share of workers in the research sector are treated as an exogenous parameter. So,

*L* = *LA* + *LY* (3)

*LA* = *sAL* (4)

It is assumed that the total number of workers grows at an exogenous rate *n*:

*L*˙

= *n* (5)

*L*

## Simplifying the Aggregate Production Function

The aggregate capital stock is defined as

*A*

Σ

*K* = *xi* (6)

*i*=1

Again, treating the savings rate as exogenous it is assumed that

*K*˙ = *sKY − δK* (7)

One observation that simplifies the analysis of the model is the fact that all of the capital goods play an identical role in the production process. For this reason, it can be assumed that the demand from producers for each of these capital goods is the same, implying that

*xi* = *x*¯ *i* = 1*,* 2*, ....A* (8)

This means that the production function can be written as

*Y* = *AL*1*−αx*¯*α* (9)

*Y*

Note now that

*K* = *Ax*¯ *⇒ x*¯ =

*K*

(10)

*A*

so output can be re-expressed as equation (11).

This looks just like the Solow model’s production function. The TFP term is written as *A*1*−α* as opposed to just *A*.

## Steady-State Growth in The Romer Model

This economy converges to a steady- state growth path in which capital and output grow at the same rate. So, we can derive the steady-state growth rate as follows. Re-write the production function as

*Y* = (*AsY L*)1*−α Kα* (12)

where

*sY* = 1 *− sA* (13)

Our usual procedure for taking growth rates give us Equation (14).

Now use the fact that the steady-state growth rates of capital and output are the same to derive that this steady-state growth rate is given by equation (15).

Finally, because the share of labour allocated to the non-research sector cannot be changing along the steady-state path (otherwise the fraction of researchers would eventually go to zero or become greater than one, which would not be feasible) we have Equation (16).

The steady-state growth rate of output per worker equals the steady-state growth rate of *A*. The only difference from the Solow model is that writing the TFP term as *A*1*−α* makes this

growth rate *A*˙ as opposed to 1 *A*˙ .

*A* 1*−α A*

## Deriving the Steady-State Growth Rate

The big difference relative to the Solow model is that the *A* term is determined within the model as opposed to evolving at some fixed rate unrelated to the actions of the agents in the model economy. To derive the steady-state growth rate in this model, note that the growth

rate of the number of capital goods is

*A*˙ *A*

= *γ* (*sA*

*L*)*λ Aφ−*1 (17)

The steady-state of this economy features *A* growing at a constant rate. This can only be the case if the growth rate of the right-hand-side of (17) is zero. Using our usual procedure for calculating growth rates of Cobb-Douglas-style items, we get Equation (18).

Again, in steady-state, the growth rate of the fraction of researchers ( *s*˙*A* ) must be zero. So,

*s*

*A*

along the model’s steady-state growth path, the growth rate of the number of capital goods

(and hence output per worker) is given by equation (19).

The long-run growth rate of output per worker in this model depends positively on three factors:

* + The parameter *λ*, which describes the extent to which diminishing marginal productivity sets in as we add researchers.
  + The strength of the “standing on shoulders” effect, *φ*. The more past inventions help to boost the rate of current inventions, the faster the growth rate will be.
  + The growth rate of the number of workers *n*. The higher this, the faster the economy.